

**Soluție**

**1.a)**  $f'(x) = \frac{-2}{(x-1)^2}$  .

**b)**  $\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1} = f'(-1)$  ;  $f'(-1) = \frac{-2}{4} = -\frac{1}{2}$  .

**c)**  $\lim_{x \rightarrow +\infty} f(x) = 1 \Rightarrow y = 1$  asimptotă orizontală spre  $+\infty$  .

**2.a)**  $\int_0^1 e^{-x} \cdot x \cdot e^x dx = \int_0^1 x dx = \frac{1}{2}$  .

**b)**  $I_1 = \int_0^1 x e^x dx = \left( x e^x - e^x \right) \Big|_0^1 = 1$  .

**c)**  $I_n = \int_0^1 x^n e^x dx = x^n e^x \Big|_0^1 - n \int_0^1 x^{n-1} e^x dx = e - n I_{n-1} \Rightarrow I_n + n I_{n-1} = e$  .