

Soluții

1. a) $A^t A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 15 \\ 15 & 45 \end{pmatrix}.$

b) $XX^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{pmatrix}.$

$$\det(XX^t) = (a^2 + c^2)(b^2 + d^2) - (ab + cd)^2 = (ad - bc)^2.$$

c) $\det(XX^t) = 0 \Rightarrow ad = bc$; obținem $\frac{a}{b} = \frac{c}{d}.$

2. a) $(x \circ y) \circ z = xyz - xy - xz - yz + x + y + z$; $x \circ (y \circ z) = xyz - xy - xz - yz + x + y + z.$

b) $x \circ y > 1 \Leftrightarrow (x-1)(y-1) > 0.$ E adevărat pentru că $x > 1, y > 1.$

c) $x \circ a = a \Leftrightarrow x(a-1) - 2(a-1) = 0, \forall x \in \mathbb{Z} \Rightarrow a = 1.$