

Rezolvare

1.a. $f'(x) = 2(x+1) + 2(x-1) = 4x$

b. $\lim_{x \rightarrow \infty} \frac{f(x)}{x^2} = \lim_{x \rightarrow \infty} \frac{2x^2 + 2}{x^2} = 2$

c. Fie $g(x) = e^{4x} - 4x - 1$, care admite $x = 0$ punct de minim $\Rightarrow g(x) \geq g(0), \forall x \in \mathbb{R}$

$$\Rightarrow g(x) \geq g(0), \forall x \in \mathbb{R} \Rightarrow 4x \leq e^{4x} - 1, \forall x \in \mathbb{R}$$

2.a. $I_1(x) = \int_0^x e^t dt = e^t \Big|_0^x = e^x - 1 \Rightarrow I_1(x) = e^x - 1.$

b. $I_2(x) = \int_0^x I_1(t) dt = e^x - x - 1.$ Deci, $\lim_{x \rightarrow -\infty} \frac{e^x - 1 - x}{x + 1} = \lim_{x \rightarrow \infty} \frac{e^{-x} + x - 1}{-x + 1} = -1$

c. Fie $I_2(x) = e^x - x - 1$, care admite $x = 0$ punct de minim $\Rightarrow I_2(x) \geq I_2(0), \forall x \in \mathbb{R}$