Soluție

1.a) $m = \lim\limits_{x\to\infty} \frac{f(x)}{x} = 1$

$l = -2$ $n = \lim\limits_{x\to\infty} (f(x) - x) = 0$

y=x asimptota oblica spre ∞

b) $f'(x) = \frac{x+1}{\sqrt[3]{(x-1)(x+2)^2}}$

f nu e derivabila pentru x $\in \{1, -2\}$

c) $\lim\limits_{x\to\infty} \frac{2arctgf(x) - \pi}{x-1}$ nedeterminarea $\frac{0}{0}$

$$l = \lim_{x\to\infty} \frac{\frac{2}{1+f^2(x)} \frac{x+1}{\sqrt[3]{(x-1)(x+2)^2}}}{-\frac{1}{x^2}}$$

$l = -2$

2.a) $I_1 = \int_1^2 (x-1)(2-x)dx$

$I_1 = \left(-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right)\Big|_1^2$

$I_1 = \frac{1}{6}$

b) Cu schimbarea x=$\frac{3}{2} - y$ obtinem $I_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} (\frac{1}{4} - x^2)^n dx$

$$I_n = \frac{1}{4} I_{n-1} + \frac{1}{2n} \int_{-\frac{1}{2}}^{\frac{1}{2}} x\left(\left(\frac{1}{4} - x^2\right)^n\right)' dx$$

de unde rezultã relatia cerutã

c) $0 \le (x-1)(2-x) \le \frac{1}{4}$, $\forall$x $\in$ [1;2]

$0 \le I_n \le \int_1^2 \left(\frac{1}{4}\right)^n = \frac{1}{4^n}$

$\lim\limits_{n\to\infty} I_n = 0$