Soluție

a) $F(x)=\int f(x)dx=\int \sqrt{x}dx=\int x^{\frac{1}{2}}dx=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}=\frac{2x\sqrt{x}}{3}+\ell; O(0,0)\in G_{F(x)}\Rightarrow F(0)=0\Rightarrow \ell=0\Rightarrow$

$F(x)=\frac{2x\sqrt{x}}{3}$

b) $\int_0^1 f(x)dx=\int_0^1 \sqrt{x}dx=\left.\frac{2x\sqrt{x}}{3}\right|_0^1=\frac{2}{3}$

c) $(f(x)-1)(f(x)+1)=(\sqrt{x}-1)(\sqrt{x}+1)=x-1\Rightarrow \int_0^2 (f(x)-1)(f(x)+1)dx=\int_0^2 (x-1)dx=\left.\left(\frac{x^2}{2}-x\right)\right|_0^2=0$

d) $\int_1^n f\left(x^2\right)dx=\left.\frac{x^2}{2}\right|_1^n=\frac{n^2-1}{2}\Rightarrow \frac{n^2-1}{2}=24\Rightarrow n_{1,2}=\pm 7; n\in\mathbb{N}\Rightarrow n=7$

e) $V_f=\pi\cdot\int_{\frac{1}{2}}^{2}\sqrt{x}^2dx=\pi\cdot\int_{\frac{1}{2}}^{2}xdx=\pi\cdot\left.\left(\frac{x^2}{2}\right)\right|_{\frac{1}{2}}^{2}=\frac{15\pi}{8}$

f) $\sqrt{x}\le\frac{x+1}{2}\Leftrightarrow 2\sqrt{x}\le x+1\Leftrightarrow 0\le\left(\sqrt{x}-1\right)^2$; pătratul oricărui număr real este pozitiv

Aplicăm teorema de monotonie:

$f(x)\ge g(x),\forall x\in[u;v]\Rightarrow \int_u^v f(x)dx\ge\int_u^v g(x)dx\Rightarrow \int_1^{2008} f(x)dx\le\int_1^{2008}\frac{x+1}{2}dx$