

Rezolvare

1. a) $f'(x) = 3^x \ln 3 - \left(\frac{1}{2}\right)^x \ln \frac{1}{2}.$

b) $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = f'(0) = \ln 6.$

c) $f'(x) = 3^x \ln 3 + \left(\frac{1}{2}\right)^x \ln 2 > 0$ pentru orice $x > 0$, deci f este crescătoare pe \mathbb{R} .

2. a) $\int f(x) dx = \frac{x^2}{2} + \ln x + C.$

b) $V(C_g) = \pi \int_1^2 \left(x + \frac{1}{x}\right)^2 dx = \pi \left(\frac{x^3}{3} + 2x - \frac{1}{x}\right) \Big|_1^2 = \frac{29\pi}{6}.$

c) $\int_1^e f(x) \ln x dx = \int_1^e x \ln x dx + \int_1^e \frac{1}{x} \ln x dx = \left(\frac{x^2}{2} \ln x - \frac{x^2}{4}\right) \Big|_1^e + \frac{\ln^2 x}{2} \Big|_1^e = \frac{e^2 + 3}{4}.$