

Soluții

1. a) $A(1,1) = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}; \det A(1,1) = 1.$

b) $A + B = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ -(b_1 + b_2) & a_1 + a_2 - (b_1 + b_2) \end{pmatrix},$ cu elemente din $\mathbb{R} \Rightarrow AB \in M.$

c) $A(0,b) = \begin{pmatrix} 0 & b \\ -b & -b \end{pmatrix}, I_2 - A(0,b) = \begin{pmatrix} 1 & -b \\ b & 1+b \end{pmatrix}; \det(I_2 - A(0,b)) = 1 + b + b^2$

$$1 + b + b^2 = \left(b + \frac{1}{2}\right)^2 + \frac{3}{4} > 0, \forall b \in \mathbb{R}.$$

2. a) $g(\hat{0}) = \hat{1}.$

b) $f = X(X^2 + \hat{2}); \hat{1}^2 = \hat{2}^2 = \hat{1}, f(\hat{0}) = f(\hat{1}) = f(\hat{2}) = \hat{0}.$

c) $h = aX^3 + bX^2 + cX + d, h(\hat{0}) = d, h(\hat{1}) = a + b + c + d, h(\hat{2}) = \hat{2}a + b + \hat{2}c + d;$

$$a + b + c = \hat{0}, a + c = \hat{0} \Rightarrow b = \hat{0}; d \in \mathbb{Z}_3.$$

Soluțiile: $X^3 + \hat{2}X, X^3 + \hat{2}X + 1, X^3 + \hat{2}x + \hat{2}, \hat{2}X^3 + X, \hat{2}X^3 + X + \hat{1}, \hat{2}X^3 + X + \hat{2}.$