Soluții

1.a) $y=0$ asimptota orizontala la ∞ si la $-\infty$.

Dreptele $x=0, x=-1$ sunt asimptote verticale.

b)Avem ca $f'(x)=\dfrac{-2\left(3x^2+3x+1\right)}{x^3.(x+1)^3}<0, \forall x\in \mathbb{R}\setminus\{-1,0\}$,de unde se obtine concluzia.

c) . $\sum_{k=1}^{n} f(k)=\sum_{k=1}^{n}\left(\dfrac{1}{k^2}-\dfrac{1}{(k+1)^2}\right)=1-\dfrac{1}{(n+1)^2}$ Limita ceruta este $\dfrac{1}{e}$

2.a) $I_1=\int_0^2\left(1-\dfrac{1}{x+1}\right)dx=2-\ln 3.$

b) Avem ca $I_n=\int_0^1\dfrac{x^n}{1+x^n}+\int_1^2\dfrac{x^n}{1+x^n}dx\le\int_0^1 x^n dx+\int_1^2 1dx=\dfrac{1}{n+1}+1.$

c)Avem ca $0\le\int_0^1\dfrac{x^n}{1+x^n}dx\le\int_0^1 x^n dx\Rightarrow 0\le\int_0^1\dfrac{x^n}{1+x^n}dx\le\dfrac{1}{n+1}$,deci $J_n=\int_0^1\dfrac{x^n}{1+x^n}dx$,are limita egala cu 0.Cum $I_n=J_n+\int_1^2\dfrac{x^n}{1+x^n}dx=J_n+\int_1^2\left(1-\dfrac{1}{1+x^n}\right)dx=J_n+1-c_n$,si c_n are limita egala cu 0,deducem ca limita ceruta este egala cu1.