Rezolvare

a) $x \perp y + \frac{(2-x)(2-y)}{2} - 2 = x + y - \frac{xy}{2} + \frac{4-2x-2y+xy}{2} - 2 = 0, \forall x, y \in \mathbb{Q}$.

$x \perp y = 2 - \frac{(2-x)(2-y)}{2}, \forall x, y \in \mathbb{Q}.$

b) Demonstrăm că: $(x \perp y) \perp z = x \perp (y \perp z), \forall x, y, z \in \mathbb{Q}$ şi că $x \perp y = y \perp x, \forall x, y \in \mathbb{Q}.$

$$(x \perp y) \perp z = \left(2 - \frac{(2-x)(2-y)}{2}\right) \perp z = \frac{\frac{(2-x)(2-y)}{2}(2-z)}{2} = \frac{(2-x)(2-y)(2-z)}{4} =$$

$x \perp (y \perp z), \forall x, y, z \in \mathbb{Q}$. Analog pentru $x \perp y$, $y \perp x, \forall x, y \in \mathbb{Q}$.

c) $e \in \mathbb{Q}$ astfel ca $x \perp e = e \perp x = x, \forall x \in \mathbb{Q}.$

Dar $x \perp e = 2 - \frac{(2-x)(2-e)}{2}, \forall x \in \mathbb{Q} \Rightarrow 2 - \frac{(2-x)(2-e)}{2} = x, \forall x \in \mathbb{Q}. \Rightarrow e = 0.$

d) Căutăm $a \in \mathbb{Q}$ astfel ca $x \perp a = a, \forall x \in \mathbb{Q}.$ Dar $x \perp a = 2 - \frac{(2-x)(2-a)}{2}, \forall x \in \mathbb{Q}$

Deci $2 - \frac{(2-x)(2-a)}{2} = a, \forall x \in \mathbb{Q}. \Rightarrow a = 2.$

e) Din punctele anterioare, $(M, \perp)$ monoid comutativ. Se demonstrează că $\forall x \in M, \exists x' \in M$ aşa încât

$x \perp x' = x' \perp x = 0$. Dar $x \perp x' = 2 - \frac{(2-x)(2-x')}{2}$

$\Rightarrow x' = 2 - \frac{4}{2-x} \in M, \forall x \in M$, deci $(M, \perp)$ grup comutativ

f) Folosind punctul d) se obține că $(-8) \perp (-7) \perp ... \perp 0 \perp 1 \perp 2 \perp ... \perp 8 = 2.$