Soluție

1.a) $h(x)=\dfrac{(x-2)+(x-1)}{(x-1)(x-2)}=\dfrac{1}{x-1}+\dfrac{1}{x-2}$.

b) $h'(x)=\dfrac{-1}{(x-1)^2}+\dfrac{-1}{(x-2)^2}\Rightarrow\dfrac{-1}{(x-1)^2}=\dfrac{1}{x-1}\Rightarrow x=0$.

c) Din $h(x)=\dfrac{f'(x)}{f(x)}\Rightarrow f'(x)=h(x)\cdot f(x)$ și $f(x)=\dfrac{f'(x)}{h(x)}$

$\Rightarrow f''(x)=h'(x)\cdot f(x)+h(x)\cdot f'(x)=h'(x)\cdot\dfrac{f'(x)}{h(x)}+\dfrac{f'(x)}{f(x)}\cdot f'(x)$.

Cum $f'(x)=2x-3\Rightarrow$ pentru $x\neq\dfrac{3}{2}$ putem împărți cu $f'(x)$ $\Rightarrow$c.c.t.d.

2.a) $V=\pi\int_1^3 x^2dx=\pi\left.\dfrac{x^3}{3}\right|_1^3=\pi\left(9-\dfrac{1}{3}\right)=\pi\cdot\dfrac{26}{3}$.

b) $\int f(x)dx=\dfrac{x^{2008}}{2008}+\dfrac{x^2}{2}+x+c$; $F(0)=1\Leftrightarrow c=1\Rightarrow F(x)=\dfrac{x^{2008}}{2008}+\dfrac{x^2}{2}+x+1$.

c) $\int_0^x f(t)\,dt=\dfrac{x^{2008}}{2008}+\dfrac{x^2}{2}+x\Rightarrow\lim\limits_{x\to\infty}\dfrac{\int_0^x f(t)dt}{x^{2008}}=\dfrac{1}{2008}$.