Soluție

a) $f'(x) = -\dfrac{1}{x^2}$.

b) $\lim\limits_{\substack{x \to 0 \\ x>0}} f(x) = \infty$, $x = 0$ asimptotă verticală la dreapta;

$\lim\limits_{x\to\infty} f(x) = 1, y = 1$ asimptotă orizontală , 2 asimptote.

c) $\lim\limits_{x\to 1} \dfrac{1-\dfrac{1}{x^2}}{x-1} = \lim\limits_{x\to 1} \dfrac{x^2-1}{x^2(x-1)} = \lim\limits_{x\to 1} \dfrac{x+1}{x^2} = 2$.

d) $\dfrac{f(\sqrt{2}) - f(1)}{\sqrt{2}-1} = -\dfrac{1}{c^2}, c^2 = \sqrt{2}, c = \sqrt{\sqrt{2}} \in (1, \sqrt{2})$

e) Fie $h:(0,\infty) \to \mathbb{R}, h(x) = f'(x)$.

$h'(x) = \dfrac{2}{x^3} > 0, \forall x > 0$, deci h strict crescătoare.

f) $f(x) = 1 + \dfrac{1}{x}$, $f\left(\dfrac{a+b}{2}\right) = 1 + \dfrac{2}{a+b}, \dfrac{f(a)+f(b)}{2} = 1 + \dfrac{a+b}{2ab}$

$\dfrac{2}{a+b} \le \dfrac{a+b}{2ab} \Leftrightarrow 4ab \le (a+b)^2 ; a, b > 0 \Leftrightarrow 0 \le (a-b)^2$