Soluție

1.a Evident $\lim_{x \searrow k} f(x) = +\infty$ si $\lim_{x \nearrow k} f(x) = -\infty$, k=1,2,...,2008

x=k asimptota verticala pentru k=1,2,...,2008

$\lim_{x \to \infty} f(x) = 0 = \lim_{x \to -\infty} f(x) \Rightarrow$ y=0 asimptota orizontala spre $+\infty$ si $-\infty$

Fie g:A $\to \mathbb{R}$, g(x)=f(x)-a

b $\lim_{x \searrow k} g(x) = \infty$, $\lim_{x \nearrow k+1} g(x) = -\infty \Rightarrow (\exists) c_k \in (k; k+1)$ astfel incat $g(c_k) = 0$

$k = 1, 2, ..., 2008$, $\lim_{x \to -\infty} g(x) = -a$, $\lim_{x \nearrow 1} g(x) = -\infty$

$\lim_{x \searrow 2008} g(x) = +\infty$, $\lim_{x \to \infty} g(x) = -a$

$\Rightarrow$ mai avem o solutie in $(-\infty;1)$ sau $(2008;\infty)$

c $f''(x) = \frac{2}{(x-1)^3} + \frac{2}{(x-2)^3} + ... + \frac{2}{(x-2008)^3}$

f'' se anuleaza in (k;k+1) o singura data(strict descrescatoare),

k=1,1,...2008

2007 puncte de inflexiune

2.a $f'(x) = e^{-x^2}$

$f'(x) > 0$ $\forall$x $\in \mathbb{R} \Rightarrow$ f strict crescatoare pe $\mathbb{R}$

b $f''(x) = -2e^{-x^2}$

$f''(x) \le 0$ $\forall$x $\in [0;\infty) \Rightarrow$ f concava pe $[0;\infty)$

$f(n) = \int_0^n e^{-t^2} dt$

c $f(n+1) - f(n) = \int_n^{n+1} e^{-t^2} dt > 0 \Rightarrow (f_n)$ crescatoare

$e^{t^2} \ge 1 + t^2 \Rightarrow f(n) \le \int_0^n \frac{1}{1+t^2} dt = arctgn < \frac{\pi}{2}$

$(f_n)_{n\ge1}$ crescator si marginit superior $\Rightarrow (f_n)_{n\ge1}$ convergent