Soluții

1.a) $\left.\begin{array}{l}\overrightarrow{OA}+\overrightarrow{OC}=\vec{0}\\ \overrightarrow{OB}+\overrightarrow{OD}=\vec{0}\end{array}\right\}\Rightarrow \overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=\vec{0}.$

Vectorul $2\vec{a}-3\vec{b}$ are coordonatele $(2x-9;6-6y)\Rightarrow$

1.b) $\begin{cases}2x-9=0\\ 6-6y=0\end{cases}\Leftrightarrow\begin{cases}x=\dfrac{9}{2}\\ y=1\end{cases}$

2.a) $\left.\begin{array}{l}\sin(C)=\dfrac{1}{2}=\dfrac{AB}{10}\Rightarrow AB=5.\\ \cos(C)=\dfrac{\sqrt{3}}{2}=\dfrac{AC}{10}\Rightarrow AC=5\sqrt{3}\end{array}\right\}\Rightarrow \sigma[ABC]=\dfrac{AB\cdot AC}{2}=\dfrac{25\sqrt{3}}{2}.$

2.b) $BC^2=AB^2+AC^2-2\cdot AB\cdot AC\cdot\cos A\Rightarrow AB^2-6\cdot AB\cdot\dfrac{1}{2}+9-16=0\Rightarrow AB^2-3AB-7=0.$

$AB=\dfrac{3+\sqrt{9+28}}{2}=\dfrac{3+\sqrt{37}}{2}.$

3.a) $N(\frac{9}{2};\frac{3}{2})\Rightarrow BN=\sqrt{\frac{25}{4}+\frac{25}{4}}=\frac{5\sqrt{2}}{2}.$

3.b)

$\begin{cases}AM=AB\\ BM=AB\end{cases}\Leftrightarrow\left\{\begin{array}{l}\sqrt{(x-2)^2+(y-1)^2}=\sqrt{8}\\ \sqrt{(x-4)^2+(y+1)^2}=\sqrt{8}\end{array}\right\}\Rightarrow -4x-2y+5=-8x+2y+17\Leftrightarrow 4x-4y-12=0\Leftrightarrow x-y-3=0\Rightarrow$

$x=y+3\Rightarrow(y+1)^2+(y-1)^2=8\Leftrightarrow y^2+1=4\Rightarrow y=\sqrt{3}\Rightarrow x=3+\sqrt{3}.$