Soluție

1.a) $\lim\limits_{x\to\infty} f(x) = 1$, y=1 asimptota orizontala spre $+\infty$

$\lim\limits_{x\to-\infty} f(x) = 1$, y=1 asimptota orizontala spre $-\infty$

$\lim\limits_{x\nearrow 0} f(x) = 0$, $\lim\limits_{x\searrow 0} f(x) = +\infty$ x=0 asimptota verticala

b)

$f''(x) = \dfrac{e^{\frac{1}{x}}(2x+1)}{x^4}$,deci x=-0,5 este punct de inflexiune

c) $\lim\limits_{x\to\infty} x^2(e^{\frac{1}{x+1}} - e^{\frac{1}{x}}) = \lim\limits_{x\to\infty} e^{\frac{1}{x}} x^2 \left(e^{-\frac{1}{x(x+1)}} - 1\right)$

$= \lim\limits_{x\to\infty} e^{\frac{1}{x}} \dfrac{-x^2}{x(x+1)} \dfrac{e^{-\frac{1}{x(x+1)}} - 1}{-\frac{1}{x(x+1)}} = -1$

2.a) $I_1 = \int_0^{\frac{\pi}{4}} tg^2 x dx$

$= \int_0^{\frac{\pi}{4}} (1 + tg^2 x) dx - \int_0^{\frac{\pi}{4}} 1 dx$

$= tgx \,|_0^{\frac{\pi}{4}} - x \,|_0^{\frac{\pi}{4}} = 1 - \dfrac{\pi}{4}$

b) $x \in \left[0; \dfrac{\pi}{4}\right] \Rightarrow tgx \in [0;1]$ $\mathrm{tg}^{2n} x > tg^{2n+2} x \Rightarrow I_{n+1} < I_n$

$I_n > 0$

$(I_n)_{n\geq 1}$ marginit inferior $\Rightarrow (I_n)$ convergent

c) $I_{n+1} = \int_0^{\frac{\pi}{4}} tg^{n+2} x dx = \int_0^{\frac{\pi}{4}} \left[tg^n x(tg^2 x + 1) - tg^n x\right] dx$

$= \dfrac{tg^{n+1} x}{n+1} \,|_0^{\frac{\pi}{4}} - I_n$; $I_{n+1} + I_n = \dfrac{1}{n+1}$

Daca $\mathrm{I_n} \to l \Rightarrow 2l = 0 \Rightarrow l = 0$