Soluție

1.a. $A \cdot B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 13 & 9 \\ 1 & -2 \end{pmatrix}$.

b. Notăm $X = \begin{pmatrix} x & y \\ t & z \end{pmatrix}$ ecuația devine $A \cdot X = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ t & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} \Rightarrow$

$\begin{pmatrix} 2x+z & 2y+t \\ -x+2z & -y+2t \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} \Rightarrow X = \begin{pmatrix} \frac{11}{5} & \frac{8}{5} \\ \frac{6}{5} & \frac{3}{5} \end{pmatrix}$.

c. $A^2 = A \cdot A = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \Rightarrow A^2 - 4A + 5I_2 = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$
$-4\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2$.

2.a. $x \circ x = 2 \Leftrightarrow 2x - 14 = 2 \Leftrightarrow x = 8$.

b. $(x \circ y) \circ z = (x + y - 14) \circ z = x + y + z - 28 = x \circ (y \circ z)$.

c. asociativitatea dem la b), elementul neutru este $e = 14$, elementele simetrizabile $x' = 28 - x$ iar comutativitatea se verifica usor