Soluție

1.a $f'(x) = nx^{n-1} + n$

$f''(x) = n(n-1)x^{n-2} \geq 0 \; \forall x \geq 0$

f convexa

b $f'(x) > 0 \; \forall x>0 \Rightarrow$ f strict crescatoare, deci injectiva pe $[0;\infty)$

f(0)=-1 si f(1)=n-2 ≥ 0, f continua $\Rightarrow (\exists)\, x_n \in (0;1]$

astfel incat $f(x_n) = 0$, x_n unic

c $f\left(\frac{1}{n}\right) = \frac{1}{n^n} > 0 \Rightarrow x_n \in \left(0;\frac{1}{n}\right)$

$\lim\limits_{n\to\infty} x_n = 0$

2.a $\int_0^1 f(x)dx = \ln\left(1+e^x\right)\Big|_0^1 = \ln\frac{e+1}{2}$

$g\left(\frac{\pi}{2}\right) = I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos t}{1+e^t}\,dt$

$g\left(\frac{\pi}{2}\right) = 1$

b $g'(x) = f(x)\cos x + f(-x)\cos x$

$g'(x) = \cos x$

c $g\left(\frac{\pi}{2}\right) = I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos t}{1+e^t}\,dt$ Fie $J=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t}\,dt$

Dar I=J si $I+J=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,dt = 2\sin t\,\Big|_0^{\frac{\pi}{2}} = 2$

$g\left(\frac{\pi}{2}\right) = 1$