Soluție

1.a) $f'(x) = \dfrac{x}{\sqrt{x^2+1}}$

$f'(x) > 0$, x>0, f′(x)<0 pentru x<0

f strict descrescatoare pe (-∞;0] si strict crescatoare pe [0;∞)

b) $\sqrt{x^2+1}f'(x) = x$

$\dfrac{x}{\sqrt{x^2+1}} f'(x) + \sqrt{x^2+1} f''(x) = 1$

$(x^2+1)f''(x) + xf'(x) = \sqrt{x^2+1}$

c) $m = \lim\limits_{x \to \infty} \dfrac{f(x)}{x} = -1$

$n = \lim\limits_{x \to -\infty} (f(x) + x) = 0$

$y = -x$ asimptota oblica spre -∞

2.a) $I_1 = \int_0^1 \dfrac{x}{x+1} dx = \int_0^1 (1 - \dfrac{1}{x+1}) dx$

$= (x - \ln(x+1)) \big|_0^1$

$= 1 - \ln 2$

b) $I_n = \int_0^1 x \dfrac{nx^{n-1}}{x^n+1} dx = \int_0^1 x \left(\ln\left(x^n+1\right)\right)' dx$

$= \ln 2 - \int_0^1 \ln\left(x^n+1\right) dx$

c) $\ln(1+t) \le t,\ \forall \mathrm{t} \ge 0$

$0 \le \int_0^1 \ln\left(1+x^n\right) dx < \int_0^1 x^n dx = \dfrac{1}{n+1}$

$deci\ \lim\limits_{n \to \infty} \int_0^1 \ln\left(1+x^n\right) dx = 0,$ de unde $\lim\limits_{n \to \infty} I_n = \ln 2$

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