

Soluție

$$\mathbf{1.a)} \quad \left. \begin{array}{l} \overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} \\ \overrightarrow{AM} = \overrightarrow{AC} + \overrightarrow{CM} \end{array} \right\} \Rightarrow 2\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{AC} \Rightarrow \overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$$

$$\mathbf{1.b)} \quad \overrightarrow{BG} = \frac{2}{3}\overrightarrow{BM} = \frac{2}{3} \cdot \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC}) = \frac{1}{3}(\overrightarrow{BA} + \overrightarrow{BC})$$

2.a)

$$BB' \perp AC \Rightarrow \sin(\widehat{BAC}) = \frac{BB'}{AB}$$

Soluția 1: $\sin(30^\circ) = \frac{BB'}{4} \Leftrightarrow \frac{1}{2} = \frac{BB'}{4} \Rightarrow BB' = 2$

$$\sigma[ABC] = \frac{AC \cdot BB'}{2} = \frac{6 \cdot 2}{2} = 6$$

Soluția 2: $\sigma[ABC] = \frac{AC \cdot AB \cdot \sin(\widehat{BAC})}{2} = \frac{4 \cdot 6 \cdot \frac{1}{2}}{2} = \frac{24}{4} = 6$

$$\mathbf{2.b)} \quad \cos A = \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC} = \frac{2(1 + \sqrt{3})}{2\sqrt{2}(1 + \sqrt{3})} = \frac{\sqrt{2}}{2} \Rightarrow m(\sphericalangle A) = 45^\circ.$$

$$\mathbf{3.a)} \quad BC = \sqrt{(x_b - x_c)^2 + (y_b - y_c)^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$\mathbf{3.b)} \quad M(1; \frac{5}{2}). \text{ Ecuația dreptei } AM : \frac{y+1}{x+1} = \frac{7}{4} \Rightarrow AM : 7x - 4y + 3$$