

Ministerul Educației, Cercetării și Tineretului
Centrul Național pentru Curriculum și Evaluare în Învățământul Preuniversitar

Rezolvare.

a) $A - B + I_2 = \begin{pmatrix} 1 & 4 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & 4 \end{pmatrix}.$

b) $2A = \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix} \Rightarrow \det(2A) = \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} = 4, \det A = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow 4 = a \cdot 1 \Rightarrow a = 4.$

c) $B^2 = \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 4 \end{pmatrix} \Rightarrow B^3 = \begin{pmatrix} 0 & 4 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -8 \\ 0 & -8 \end{pmatrix} = 4 \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} = 4B.$

d) $\begin{pmatrix} 1 & x \\ y & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2+x \\ y & 2y+1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2+x \\ y & 2y+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{cases} x = -2 \\ y = 0 \end{cases}.$ Pentru $x = -2, y = 0$ avem
 $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ este inversa matricei A , sau $A^{-1} = \frac{1}{\det A} A^* \Rightarrow A^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \Rightarrow x = -2, y = 0.$

e) $X \in \mathcal{M}_2(\mathbb{R}) \Rightarrow X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}, x, y, z, t \in \mathbb{R}, A \cdot X = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x+2z & y+2t \\ z & t \end{pmatrix} \Rightarrow$
 $\begin{pmatrix} x+2z & y+2t \\ z & t \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} \Rightarrow \begin{cases} x+2z=0 \\ y+2t=-2 \\ z=0 \\ t=-2 \end{cases} \Rightarrow x=0, y=2, z=0, t=-2 \Rightarrow X = \begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix}, \text{ sau } X = A^{-1}B.$

f) $A + B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -2 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, (A+B)^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2,$

$$(A+B)^3 = (A+B)^2 (A+B) = A+B, (A+B)^4 = (A+B)^2 (A+B)^2 = I_2 \Rightarrow$$

$$A+B+(A+B)^2+(A+B)^3+(A+B)^4 = 2 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + 2I_2 = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}.$$