

Soluții

$$\mathbf{1.a)} \quad \left. \begin{aligned} \overrightarrow{MO} &= \frac{1}{2}(\overrightarrow{MA} + \overrightarrow{MC}) \\ \overrightarrow{MO} &= \frac{1}{2}(\overrightarrow{MB} + \overrightarrow{MD}) \end{aligned} \right\} \Rightarrow \frac{1}{2} \cdot (\overrightarrow{MA} + \overrightarrow{MC} - \overrightarrow{MB} - \overrightarrow{MD}) = \vec{0} \Rightarrow \overrightarrow{MA} + \overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{MD}$$

$$\mathbf{1.b)} \quad 3\vec{a} + 4\vec{b} = (9 + 4\alpha)\vec{i} + (12 - 4\beta)\vec{j} = \vec{0} \Rightarrow \alpha = -\frac{9}{4}, \beta = 3$$

$$AC^2 + BC^2 = 144 + 25 = 169 = 13^2 = AB^2 \Rightarrow \triangle ABC \text{ este dreptunghic în } A.$$

$$\mathbf{2.a)} \quad \sigma[ABC] = \frac{AC \cdot AB}{2} = 30$$

$$\mathbf{2.b)} \quad \cos A = \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC} = \frac{1 + 3 + 2\sqrt{3} + 2 - 4}{2\sqrt{2}(1 + \sqrt{3})} = \frac{2(1 + \sqrt{3})}{2\sqrt{2}(1 + \sqrt{3})} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\text{Deci } \cos A = \frac{\sqrt{2}}{2} \Rightarrow m(\sphericalangle A) = 45^\circ.$$

$$\mathbf{3.a)} \quad BC = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2} = \sqrt{16 + 1} = \sqrt{17}.$$

$$\text{Dacă } N \text{ este mijlocul lui } [AC] \Rightarrow N\left(1; \frac{1}{2}\right).$$

$$\mathbf{3.b)} \quad \text{Ecuația dreptei } BN \text{ este: } \frac{y - y_B}{y_B - y_N} = \frac{x - x_N}{x_B - x_N} \Leftrightarrow \frac{y - 3}{3 - \frac{1}{2}} = \frac{x - 1}{-2} \Leftrightarrow -2y + 6 = \frac{5x - 5}{2} \Rightarrow$$

$$BN : 5x + 4y - 17 = 0$$