

Soluție

1.a. Determinantul $\begin{vmatrix} \sqrt{2008}-1 & -1 \\ 1 & \sqrt{2008}+1 \end{vmatrix} = (\sqrt{2008}-1)(\sqrt{2008}+1)+1=2008$ deoarece $i^2 = -1$.

b. Determinantul $D = \begin{vmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{vmatrix} = x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 4^2 - 2 = 14$, unde am ținut cont de relațiile lui Viète.

c. $A^2 = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, A^3 = A^2 \cdot A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$

$$\Rightarrow A^3 + A^2 + A = 0_2.$$

2.a. $x \circ y = 2xy - 8x - 8y + 36 = 2xy - 8x - 8y + 32 + 4 = 2x(y-4) - 8(y-4) + 4 = 2(x-4)(y-4) + 4$.

b. Ecuația se mai scrie $2(x-4)(x-4) + 4 = 36 \Leftrightarrow (x-4)^2 = 16 \Leftrightarrow x_1 = -4, x_2 = 12$.

c. $\sqrt{1} \circ \sqrt{2} \circ \sqrt{3} \circ \dots \circ \sqrt{2008} = \sqrt{1} \circ \sqrt{2} \circ \sqrt{3} \circ \dots \circ \sqrt{16} \circ \dots \circ \sqrt{2008} = a \circ 4 \circ b = 4$.