

Soluție

$$1.a) \left. \begin{array}{l} \overrightarrow{MA} = \overrightarrow{MO} + \overrightarrow{OA} \\ \overrightarrow{MC} = \overrightarrow{MO} + \overrightarrow{OC} \end{array} \right\} \Rightarrow \overrightarrow{MA} + \overrightarrow{MC} = 2\overrightarrow{MO} = \overrightarrow{MB} + \overrightarrow{MD}.$$

$$1.b) \overrightarrow{AB} = 3\overrightarrow{AC} \Leftrightarrow (x-2; 2y-4) = (-9; -15) \Rightarrow x = -7; y = -\frac{11}{2}.$$

$$\triangle ABC \text{ este dreptunghic isoscel} \Rightarrow 2AB^2 = 64 \Rightarrow AB = 4\sqrt{2}.$$

$$2.a) \sigma[ABC] = \frac{AB \cdot AC}{2} = \frac{16 \cdot 2}{2} = 16.$$

$$\begin{cases} m(\sphericalangle B) + m(\sphericalangle C) = 90^\circ \cdot 2 \\ 3m(\sphericalangle B) + 2m(\sphericalangle C) = 225^\circ \end{cases} \Leftrightarrow \begin{cases} 2m(\sphericalangle B) + 2m(\sphericalangle C) = 180^\circ \\ 3m(\sphericalangle B) + 2m(\sphericalangle C) = 225^\circ \end{cases}$$

$$2.b) \quad m(\sphericalangle B) = 45^\circ \Rightarrow \triangle ABC \text{ este dreptunghic isoscel.}$$

$$\sigma[ABC] = \frac{AB \cdot AC}{2} = \frac{AC^2}{2} = \frac{25}{2}.$$

$$3.a) AB: \frac{x-x_A}{x_B-x_A} = \frac{y-y_A}{y_B-y_A} \Leftrightarrow \frac{x}{1} = \frac{y+3}{3} \Rightarrow AB: 3x - y - 3 = 0.$$

Fie $OD \perp AB, D \in [AB]$.

$$3.b) OD = \frac{OA \cdot OB}{AB} = \frac{\sqrt{9} \cdot 1}{\sqrt{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}.$$