Soluție

1.a $\lim\limits_{x\to\infty} f(x) = \dfrac{\pi}{2}$

$y = \dfrac{\pi}{2}$ asimptota orizontala spre $+\infty$

b $g'(x) = f'(x+1) - f'(x) - f'\left(\dfrac{1}{x^2+x+1}\right)\cdot\left(\dfrac{1}{x^2+x+1}\right)'$

$g'(x) = 0$

g derivabila pe $\mathbb{R}$ $\Rightarrow$ g(const)=g(0)=0 $\forall$x

c $arctg\,\dfrac{1}{k^2+k+1} = arctg(k+1) - arctgk$

$\sum\limits_{k=1}^{n} arctg\,\dfrac{1}{k^2+k+1} = arctg(n+1) - \dfrac{\pi}{4}$

$\lim\limits_{n\to\infty}(arctg(n+1) - \dfrac{\pi}{4}) = \dfrac{\pi}{2}$

2.a $I_1 = \int_0^1 e^{-x} x dx$

$= \int_0^1 x(e^{-x})' dx = -xe^{-x}\,|_0^1 + \int_0^1 e^{-x} dx =$

$= -e^{-1} - e^{-x}\,|_0^1 = -2e^{-1} + 1$

b $I_n = \int_0^1 \left(-e^{-x}\right)' x^n dx = -x^n e^{-x}\,|_0^1 + n\int_0^1 e^{-x} x^{n-1} dx$

$= -e^{-1} + nI_{n-1}$

c sirul $(\mathrm{I_n})_{n\geq 1}$ este descrescator

sirul $(\mathrm{I_n})_{n\geq 1}$ marginit inferior de 0, deci convergent

Fie l= $\lim\limits_{x\to\infty} x_n$. Daca l>0, trecand la limita obtinem l=∞, absurd, deci l=0.