

Soluție

1. $a_{n+1} - a_n = \frac{4(n+1)}{n+4} - \frac{4n}{n+3} = \frac{12}{(n+3)(n+4)} \Rightarrow a_{n+1} - a_n > 0, \forall n \in \mathbb{N} \Rightarrow$ șirul este crescător .

2. $x^2 + x + 1 = -x^2 - 2x + 6 \Rightarrow 2x^2 + 3x - 5 = 0 \Rightarrow x \in \left\{ -\frac{5}{2}, 1 \right\} \Rightarrow A\left(-\frac{5}{2}, \frac{19}{4} \right), B(1, 3)$

3. $x - \frac{\pi}{4} = 3x + \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z} \Rightarrow x \in \left\{ \frac{3\pi}{4}; \frac{7\pi}{4} \right\},$
 $x - \frac{\pi}{4} = -3x - \frac{\pi}{4} + (2k+1)\pi, k \in \mathbb{Z} \Rightarrow x \in \left\{ \frac{\pi}{4}; \frac{3\pi}{4}; \frac{5\pi}{4}; \frac{7\pi}{4} \right\}.$

4. $2^n = 32 \Rightarrow n = 5, T_4 = C_5^3 (2x^2)^2 (-5y)^3 \Rightarrow T_4 = -5000x^4 y^3.$

5. $\frac{m}{2} = \frac{3}{n} = \frac{2}{-8} \Rightarrow (m, n) = \left(-\frac{1}{2}, -12 \right).$

6. $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0 \Rightarrow AC \perp BD \Rightarrow AB^2 = OB^2 + OA^2, CD^2 = OD^2 + OC^2,$
 $AD^2 = OD^2 + OA^2, BC^2 = OC^2 + OB^2 \Rightarrow AB^2 + CD^2 = AD^2 + BC^2.$