

Soluții

1. a) $M_1 + M_2 = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}; \det(M_1 + M_2) = 4.$

b) $M_a^2 = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix}.$

c) $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}, M_a X = \begin{pmatrix} x + az & y + at \\ z & t \end{pmatrix}, XM_a = \begin{pmatrix} x & ax + y \\ z & az + t \end{pmatrix};$

$x + az = x, y + at = ax + y, z = z, t = az + t, \forall a \in \mathbb{R}$; obținem $z = 0, t = x$, deci $X = \begin{pmatrix} x & y \\ 0 & x \end{pmatrix}$, pentru $x, y \in \mathbb{R}$, oarecare.

2. a) $x * 0 = x.$

b) $x * (y * z) = x * \sqrt[3]{y^3 + z^3} = \sqrt[3]{x^3 + y^3 + z^3}; (x * y) * z = \sqrt[3]{x^3 + y^3} * z = \sqrt[3]{x^3 + y^3 + z^3}.$

c) $x_1 = \sqrt[3]{2x_0^3} = \sqrt[3]{2}x_0$. Prin inducție: $x_n = \sqrt[3]{n+1} \cdot x_0$; $x_7 = \sqrt[3]{8}x_0 = 2x_0 \in \mathbb{Q}$, pentru $x_0 \in \mathbb{Q}$.