Soluție

1.a)

$f'(x) = 3e^{3x} + 2$, f(0)=2, f'(0)=5

y-2=5x

b) f'(x)>0 $\Rightarrow$ f strict crescãtoare $\Rightarrow$ f injectivã

$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} = -\infty$, f continua $\Rightarrow$ f surjectiva

f bijectiva $\Rightarrow$ f inversabila

c) Suma este egala cu $\frac{1}{e^3-1}$ $e^{-3}+e^{-6}+...+e^{-3n} = e^{-3}\frac{1-e^{-3n}}{1-e^{-3}}$ avand limita $\frac{1}{e^3-1}$.

2.a) $a_1 = \int_0^1 \sin \pi x dx$

$a_1 = \frac{2}{\pi}$

b) $\sin \pi x < 1 \Rightarrow a_{n+1} = \int_0^{a_n} \sin \pi t dt < \int_0^{a_n} 1 dt = a_n$

$a_{n+1} = \int_0^{a_n} f(t)dt > 0$

$(a_n)_{n\geq 0}$ descrescator si marginit inferior $\Rightarrow (a_n)_{n\geq 0}$ convergent

c) $(a_n)_{n\geq 0}$ convergent in x= $\lim_{n\to\infty} a_n$

si obtinem, prin trecere la limita, x=$\int_0^{x} \sin \pi t dt$

$g:\mathbb{R}\to\mathbb{R}$, g(x)=x-$\int_0^{x} \sin \pi t dt$, g(0)=0

$g'(x) = 1 - \sin \pi x \geq 0 \Rightarrow x = 0$ solutie unica