

Soluție

1. a) $12\overrightarrow{CE} = 4\overrightarrow{AC}$.

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AD} + \frac{1}{4}\overrightarrow{DB} = \overrightarrow{AD} + \frac{1}{4}\overrightarrow{DA} + \frac{1}{4}\overrightarrow{AB} = \frac{3\overrightarrow{AD} + \overrightarrow{AB}}{4} \Rightarrow 4\overrightarrow{AC} = 3\overrightarrow{AD} + \overrightarrow{AB} \Rightarrow$$

$$12\overrightarrow{CE} = 3\overrightarrow{AD} + \overrightarrow{AB}.$$

1. b) $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} = 3\overrightarrow{CE} + 3\overrightarrow{DC} = 3(\overrightarrow{CE} + \overrightarrow{DC}) = 3\overrightarrow{DE}$

2. a) În triunghiul ABC $m(\sphericalangle A) = 90^\circ$, $m(\sphericalangle B) = 60^\circ \Rightarrow AC = AB\sqrt{3} = 5\sqrt{3} \Rightarrow \mathcal{A}_{ABC} = \frac{AB \cdot AC}{2} = \frac{25\sqrt{3}}{2}$

2. b) Din teorema sinusurilor în triunghiul DBC avem: $\frac{CD}{\sin(\sphericalangle DBC)} = \frac{BD}{\sin C}$.

$$CD = AC - AD = 5(\sqrt{3} - 1). \text{ Din triunghiul dreptunghic și isoscel } ABD \text{ avem } BD = AB\sqrt{2} = 5\sqrt{2}$$

$$m(\sphericalangle C) = 90^\circ - 60^\circ = 30^\circ.$$

$$\text{Deci } \frac{5(\sqrt{3}-1)}{\sin(\sphericalangle DBC)} = \frac{5\sqrt{2}}{\frac{1}{2}} \Rightarrow \sin(\sphericalangle DBC) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

3. a) $AB: \frac{y-y_A}{x-x_A} = \frac{y_B-y_A}{x_B-x_A} \Rightarrow AB: \frac{y}{x-12} = -\frac{5}{12} \Rightarrow AB: 12y+5x-60=0$

3. b) $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \Rightarrow AB = \sqrt{12^2 + 5^2} = 13.$