

Soluție

1. Avem $2 < 3 < 4 \Rightarrow \log_2 2 < \log_2 3 < \log_2 4 \Rightarrow 1 < \log_2 3 < 2 \Rightarrow \log_2 3 \in (1, 2)$.

2. $x^2 + 3x + m > 0$, oricare ar fi $x \in \mathbb{R} \Leftrightarrow \Delta < 0 \Leftrightarrow 9 - 4m < 0 \Leftrightarrow m > \frac{9}{4} \Leftrightarrow m \in \left(\frac{9}{4}, \infty\right)$.

3. Avem $\sin x + \cos x = \sin x + \sin\left(\frac{\pi}{2} - x\right) = 2 \sin \frac{\pi}{4} \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right)$

Ecuția devine $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Leftrightarrow x - \frac{\pi}{4} = \pm \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$

Mulțimea soluțiilor ecuației inițiale este: $\{2k\pi / k \in \mathbb{Z}\} \cup \left\{\frac{\pi}{2} + 2k\pi / k \in \mathbb{Z}\right\}$.

4. $\forall n \in \mathbb{N}, n \geq 3$ avem $C_n^2 + C_n^3 = \frac{n!}{2!(n-2)!} + \frac{n!}{3!(n-3)!} = \frac{3n! + (n-2)n!}{3!(n-2)!} = \frac{n!(n+1)}{3!(n-2)!} = \frac{(n+1)!}{3!(n-2)!} = C_{n+1}^3$.

5. Avem $d_1 \cap d_2 = \{A(1; -1)\}$. Atunci $A \in d_3 \Leftrightarrow 1 - 1 + a = 0 \Leftrightarrow a = 0$.

6. Din teorema cosinusului, $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A \Rightarrow BC = \sqrt{13}$.

Perimetrul triunghiului ABC este $AB + BC + AC = 7 + \sqrt{13}$.