Rezolvare.

a) $\det A = \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1$ număr impar $\Rightarrow A \in M$.

b) $A - 2I_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -2 \end{pmatrix}$.

c) $A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow A^3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$.

d) $A^3 = I_3 \Rightarrow A^2 A = AA^2 \Rightarrow A^{-1} = A^2 \Rightarrow \det A^{-1} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1$ număr impar $\Rightarrow A^{-1} \in M$.

e) $\begin{vmatrix} 2a-1 & a & 2 \\ -1 & a+1 & 0 \\ 2 & 1 & 1 \end{vmatrix} = 2a^2 + 2a - a - 1 - 2 - 4a - 4 + a = 2a^2 - 2a - 7 = 2\left(a^2 - a\right) - 7$. Expresia $2\left(a^2 - a\right)$ reprezintă un număr par oricare ar fi $a \in \mathbb{Z}$ $\Rightarrow 2\left(a^2 - a\right) - 7$ este un număr impar $\Rightarrow B \in M$.

f) $Y \in \mathcal{M}_3(\mathbb{R}) \Rightarrow Y = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \Rightarrow YA = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} b & c & a \\ e & f & d \\ h & i & g \end{pmatrix}$,

$AY = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix} \Rightarrow YA = AY \Rightarrow b = g = f,\ a = e = i,\ c = d = h \Rightarrow Y = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$,
$a,\ b,\ c \in \mathbb{R}$.