Soluție

a) Prin calcul direct rezultă $f(x)-\frac{1}{2x-1}+\frac{1}{x+2}=0, \forall x\in\mathbb{R}-\left\{-2,\frac{1}{2}\right\}$

b) $f'(x)=\frac{-2}{(2x-1)^2}+\frac{1}{(x+2)^2}=\frac{2x^2-12x-7}{(2x-1)^2(x+2)^2}, x\in\mathbb{R}-\{1,-2\}$

c) $y-f(1)=f'(1)(x-1)$

$y-\frac{2}{3}=-\frac{17}{9}(x-1)\Rightarrow 9y+17x-23=0$

d) $\lim\limits_{x\to\pm\infty} f(x)=0\Rightarrow y=0$ asimptotă orizontală spre $\pm\infty$

$\Rightarrow$ graficul funcției nu are asimptotă oblică

$l_s(-2)=\lim\limits_{x\nearrow-2} f(x)=\infty$

$l_d(-2)=\lim\limits_{x\searrow-2} f(x)=-\infty\Rightarrow x=-2$ asimptotă verticală

$l_s\left(\frac{1}{2}\right)=\lim\limits_{x\nearrow\frac{1}{2}} f(x)=-\infty$

$l_d\left(\frac{1}{2}\right)=\lim\limits_{x\searrow\frac{1}{2}} f(x)=\infty\Rightarrow x=\frac{1}{2}$ asimptotă verticală

e) $\lim\limits_{x\to5}\frac{x-5}{\sqrt{2x-1}-3}=\lim\limits_{x\to5}\frac{\sqrt{2x-1}+3}{2}=3$

f) g continuă în $x=1\Rightarrow l_s(1)=l_d(1)=g(1)\Rightarrow 4+a=b+1\Rightarrow a=b-3$

g derivabilă în $x=1\Rightarrow g_s'(1)=g_d'(1)\Rightarrow \lim\limits_{x\nearrow1}\frac{g(x)-g(1)}{x-1}=\lim\limits_{x\searrow1}\frac{g(x)-g(1)}{x-1}$

$\lim\limits_{x\nearrow1}\frac{x^2+3x+a-4-a}{x-1}=\lim\limits_{x\searrow1}\frac{bx+1-4-a}{x-1}$

$\lim\limits_{x\nearrow1}\frac{(x-1)(x+4)}{x-1}=\lim\limits_{x\searrow1}\frac{bx-3-b+3}{x-1}$

$\Rightarrow b=5$

$\Rightarrow a=2$