Rezolvare

1.a. Avem $A \cdot B = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

b. Din $A^2 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A^3 = A^2 \cdot A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_2$

avem $A^2 + A^3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

c. Notăm $X = \begin{pmatrix} x & y & z \\ t & e & f \\ g & h & l \end{pmatrix}$, atunci $A \cdot X = \begin{pmatrix} t+g & e+h & f+l \\ g & h & f \\ 0 & 0 & 0 \end{pmatrix}$ şi $X \cdot A = \begin{pmatrix} 0 & x & x+y \\ 0 & t & t+e \\ 0 & g & g+h \end{pmatrix}$ şi egalând cele două avem $g = 0, h = 0, t = h = 0,\ f = y = 0, l = e = x,$ deci X este de forma $X = \begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}$.

2.a. Dezvoltând relaţia $f(-1) + f(1) = 2a + 1 \Leftrightarrow 2a + 2c = 2a + 1 \Leftrightarrow c = \frac{1}{2}$.

b. $f(x) = 0 \Leftrightarrow x^3 - 3x^2 + x + 1 = 0 \Leftrightarrow (x-1)\left(x^2 - x + 1\right) = 0$ care are rădăcinile $x_1 = 1,\ x_2 = \frac{1 - i\sqrt{3}}{2}$ şi $x_3 = \frac{1 + i\sqrt{3}}{2}$.

c. $\begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix} = \begin{vmatrix} -a & -a & -a \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix} = -a \begin{vmatrix} 1 & 0 & 0 \\ x_2 & x_3 - x_2 & x_1 - x_2 \\ x_3 & x_1 - x_3 & x_2 - x_3 \end{vmatrix} = -a\left(-a^2 + 3b\right) = a^3 - 3ab.$ (s-au adunat liniile 2 şi 3 la linia 1,apoi s-a scăzut coloana 1 la 2 şi la 3).