Soluție

1.

a) $A_{99}=\left\{x=\dfrac{k+5}{k+1} \middle| k\le 99,\ \text{unde } k\in\mathbb{N}\right\}$.Avem

$\dfrac{k+5}{k+1}=1+\dfrac{4}{k+1}$. $\Rightarrow A_{99}=\left\{1+\dfrac{4}{1},1+\dfrac{4}{2},...1+\dfrac{4}{100}\right\}$

A_{99} are 100 elemente.

b) $1+\dfrac{4}{k+1}\in \mathbf{N}$. $k+1$ divizor natural al lui $4\Leftrightarrow k\in\{0,1,3\}$

$1+\dfrac{4}{1}=5;1+\dfrac{4}{2}=3;1+\dfrac{4}{4}=2$

2. $a_2=-\dfrac{3}{2};\ a_3=\dfrac{8}{3};\ a_4=-\dfrac{15}{4}$; $a_5=-\dfrac{15}{4}$.

$10\cdot a_2\cdot a_3\cdot a_4\cdot a_5=10\cdot 3\cdot 8\cdot 3=1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6=6!$

3. Dupã prima scumpire: $2000+\dfrac{5}{100}\cdot 2000=2100$. Dupã a doua scumpire: $2100+\dfrac{5}{100}\cdot 2100=2205$

4. **a)** $f(5)=25-\sqrt{2}$, $f(9)=45-\sqrt{2}$, $\dfrac{f(5)-f(9)}{5-9}=5$

b) $a=\dfrac{\sqrt{2}}{5}\in\mathbb{R}-\mathbb{Q}$; $f(a)=0\in\mathbb{Q}$