

Soluții

$$1.a) \left. \begin{array}{l} \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} \\ \overrightarrow{AF} + \overrightarrow{FE} + \overrightarrow{ED} = \overrightarrow{AD} \end{array} \right\} \Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AF} + \overrightarrow{FE} + \overrightarrow{ED}.$$

$$1.b) \left. \begin{array}{l} \overrightarrow{BG} = \frac{2}{3} \overrightarrow{BM}, \text{ M mijlocul segmentului } [AC]. \\ \overrightarrow{BM} = \frac{1}{2} (\overrightarrow{BA} + \overrightarrow{BC}) \end{array} \right\} \Rightarrow \overrightarrow{BG} = \frac{1}{3} (\overrightarrow{BA} + \overrightarrow{BC}),$$

$\triangle ABC$ este dreptunghic isoscel.

$$2.a) AB = AC = BC \cdot \sin(\sphericalangle B) = 27 \cdot \frac{\sqrt{2}}{2} \Rightarrow \sigma[ABC] = \frac{AB \cdot AC}{2} = \frac{\frac{729}{4} \cdot 2}{2} = \frac{729}{4}.$$

$$2.b) BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A \Rightarrow AC^2 - 2AC + 1 = 0 \Rightarrow AC = 1.$$

$$3.a) M\left(\frac{3}{2}; 3\right) \Rightarrow AM = \sqrt{\left(\frac{3}{2} + 2\right)^2 + (3 + 3)^2} = \sqrt{\frac{49}{4} + 36} = \frac{\sqrt{193}}{2}.$$

$$3.b) m_{BC} = -\frac{4}{5}. \text{ Ecuația dreptei cerute satisface: } \frac{y-3}{x+2} = -\frac{4}{5} \Leftrightarrow -4x-8=5y-15 \Rightarrow d: 4x+5y-7=0.$$