

Soluții

$$1.a) \overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}, \overrightarrow{ND} = \frac{1}{2}\overrightarrow{AB}, \overrightarrow{PM} = \frac{1}{2}\overrightarrow{CA}$$

$$\overrightarrow{MN} + \overrightarrow{NP} + \overrightarrow{PM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}) = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{CA}) = \frac{1}{2} \cdot \vec{0} = \vec{0}$$

$$1.b) \overrightarrow{AB} = 3\vec{i} - 3\vec{j}$$

$$\text{Fie } D(a, b). \text{ Atunci } \overrightarrow{CD} = (a-4)\vec{i} + (b-2)\vec{j}$$

$$3 = 2(a-4) \Rightarrow a = \frac{7}{2}$$

$$-3 = 2(b-2) \Rightarrow b = \frac{1}{2}$$

$$\overrightarrow{AB}(3; -3)$$

$$2.a) \left. \begin{array}{l} AB^2 = 4 + 9 = 13 \\ BC^2 = 9 + 4 = 13 \\ AC^2 = 25 + 1 = 26 \end{array} \right\} \Rightarrow AB^2 + BC^2 = AC^2 \Rightarrow \triangle ABC \text{ este dreptunghic în } B.$$

$$2.b) BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A = 16 + 2 - 8\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 10 \Rightarrow BC = \sqrt{10}$$

$$3.a) M\left(\frac{3}{2}; -3\right), \text{ unde } M \text{ este mijlocul segmentului } [BC].$$

$$AM : \frac{\frac{x-1}{3}-1}{2} = \frac{y+2}{-3+2} \Rightarrow -x+1 = \frac{1}{2}(y+2) \Rightarrow AM : 2x + y = 0$$

$$3.b) AB : \frac{x-\frac{5}{2}}{-\frac{5}{2}} = \frac{y}{\frac{5}{3}} \Leftrightarrow -2x+5 = 3y \Rightarrow AB : 2x + 3y - 5 = 0.$$

$$\text{Coordonatele lui } C, x=1, y=1 \text{ verifică ecuația dreptei } AB : 2 + 3 - 5 = 0.$$