Soluții

1.a) Se dem. prin inductie.

b) $x_{n+1} - x_n = \dfrac{x^5{}_n - x_n}{4}$. Demonstreaza ca $x_{n+1} - x_n < 0$. Sirul fiind descrescator si marginit este convergent**.**

c) Avem ca $\lim\limits_{n\to\infty} x_n = 0$ $\lim\limits_{n\to\infty} \dfrac{x_{n+1}}{x_n} = \lim\limits_{n\to\infty} \dfrac{x^4{}_n + 3}{4} = \dfrac{3}{4}$.

Din $\dfrac{x_{n+2}}{x_n} = \dfrac{x_{n+2}}{x_{n+1}} \cdot \dfrac{x_{n+1}}{x_n}$ si relatia anterioara, se deduce ca limita ceruta

este $\left(\dfrac{3}{4}\right)^2$.

2.a) Aria ceruta este $A = \int_0^1 f_1(x)\,dx = arctgx\Big|_0^1 = \dfrac{\pi}{4}$.

b) $I = \int_0^1 \dfrac{x}{\left(1+x^2\right)^2} dx = \dfrac{1}{2}\int_0^1 \left(1+x^2\right)' \left(1+x^2\right)^{-2} dx = \dfrac{-1}{2\left(x^2+1\right)}\Bigg|_0^1 = \dfrac{1}{4}$.

c) Sirul ce ne intereseaza se scrie $a_n = n \cdot \sum\limits_{k=1}^{n} \dfrac{1}{n^2 + k^2} = \dfrac{1}{n}\sum\limits_{k=1}^{n} \dfrac{1}{1 + \dfrac{k^2}{n^2}}$

Arata ca $\lim\limits_{n\to\infty} a_n = \int_0^1 \dfrac{1}{1+x^2} dx = \dfrac{\pi}{4}$.