Rezolvare

1.a. Din $A\cdot B=\begin{pmatrix}4&-2&-2\\-2&4&-2\\-2&-2&4\end{pmatrix}\cdot\begin{pmatrix}-2&-2&-2\\-2&-2&-2\\-2&-2&-2\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}=O_3$

b.

$A^2=\begin{pmatrix}4&-2&-2\\-2&4&-2\\-2&-2&4\end{pmatrix}\cdot\begin{pmatrix}4&-2&-2\\-2&4&-2\\-2&-2&4\end{pmatrix}=\begin{pmatrix}24&-12&-12\\-12&24&-12\\-12&-12&24\end{pmatrix}=6A.$

$B^2=\begin{pmatrix}-2&-2&-2\\-2&-2&-2\\-2&-2&-2\end{pmatrix}\cdot\begin{pmatrix}-2&-2&-2\\-2&-2&-2\\-2&-2&-2\end{pmatrix}=\begin{pmatrix}12&12&12\\12&12&12\\12&12&12\end{pmatrix}=-6A.$

c. $C^n=(A+B)^3=C_3^0A^3+C_3^1A^2B+C_3^2AB^2+C_3^3B^3=A^3+B^3=$

$A^2A+B^2B=6AA-6BB=6\cdot 6A-6\cdot(-6)B=6^2(A+B),$ deoarece $A\cdot B=O_3$.

2.a. $x\circ y=xy+2x+2y+2=xy+2x+2y+4-2=x(y+2)+2(y+2)-2=(x+2)(y+2)-2.$

b. $x*e=e*x=x\Leftrightarrow x+e+2=x\Leftrightarrow e=-2$ pentru prima lege și

$x\circ e=e\circ x=x\Leftrightarrow xe+2x+2e+2=x\Leftrightarrow e(x+2)=-x-2\Leftrightarrow e=\dfrac{-x-2}{x+2}=-1$ pentru a doua lege.

c. Sistemul se mai scrie

$$\begin{cases}x^2+y^2+2=7\\(x^2+2)(y^2+2)-2=16\end{cases}\Leftrightarrow\begin{cases}x^2+y^2=5\\(x^2+2)(y^2+2)=18\end{cases}\Leftrightarrow\begin{cases}x=1\\y=2\end{cases},\begin{cases}x=-1\\y=-2\end{cases},\begin{cases}x=2\\y=1\end{cases},\begin{cases}x=-2\\y=-1\end{cases}.$$