

Soluție:

$$\text{a) } a=1, b=0 \Rightarrow A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \det(A) = -1$$

$$\det(A) + \det(I_3) = -1 + 1 = 0$$

$$\text{b) } A^2 = \begin{pmatrix} a^2 + b^2 & 0 & 0 \\ 0 & a^2 + b^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{c) } aA + bI_3 = O_3 \Leftrightarrow \begin{pmatrix} a^2 + b & ab & 0 \\ ab & -a^2 + b & 0 \\ 0 & 0 & a + b \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow a = b = 0$$

$$\text{d) } A \text{ neinvertibilă} \Leftrightarrow \det(A) = 0 \Leftrightarrow -a^2 - b^2 = 0 \Leftrightarrow a = b = 0$$

$$\text{e) } A^{-1} = A \Leftrightarrow A^2 = I_3 \Leftrightarrow a^2 + b^2 = 1$$

$$a^2 + b^2 = 1, a, b \in \mathbb{Z} \Leftrightarrow a = \pm 1, b = 0 \text{ sau } a = 0, b = \pm 1$$

$$\text{f) } A^2 = I_3 \Leftrightarrow a^2 + b^2 = 1$$

$$a = \frac{1}{2} \Rightarrow b^2 = \frac{3}{4} \Rightarrow b = \pm \frac{\sqrt{3}}{2}$$