Rezolvare

1) **a)** $f'(x)=\frac{1}{(x+2)^2+1}-\frac{1}{x^2+1}.$

b) $x+2>x\Rightarrow \operatorname{arctg}(x+2)>\operatorname{arctg}x\Rightarrow f(x)>0.$

f este strict crescătoare pe intervalul $(-\infty,-1)$ şi strict descrescător pe intervalul $(1,+\infty)$, deci -1 este maxim global. $f(-1)=\operatorname{arctg}1-\operatorname{arctg}(-1)=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}.$

c) Se arată că $g'(x)=0,\ \forall x\in\mathbb{R}\Rightarrow g$ este constantă.

2) **a)** $\int_1^2 \frac{x^2-1+\frac{1}{1+x^2}}{x}dx=\frac{x^2}{2}\Big|_1^2-\ln x\Big|_1^2-\frac{1}{2}\ln\left(x^2+1\right)\Big|_1^2=\frac{3}{2}-\frac{1}{2}\ln 5+\frac{1}{2}\ln 2.$

b) $f(x)\geq\frac{x^3}{3}-x-\frac{\pi}{2}\rightarrow\int_0^x f(t)dt\geq\frac{x^4}{12}-\frac{x^2}{2}-\frac{\pi}{2}\cdot x.$ Deci $\lim\limits_{x\to\infty}\frac{\int_0^x f(t)dt}{x^3}=+\infty.$

c) $g(x)\geq f(x),\ \forall x\in[0,1)\Rightarrow \mathrm{A}=\int_0^1\left(g(x)-f(x)\right)dx=\left(\frac{x^2}{2}-\frac{x^4}{12}\right)\Big|_0^1=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}.$