Soluții

1.a) $\overrightarrow{AG}=\frac{2}{3}\overrightarrow{AM}$. M mijlocul segmentului $[BC]$.

$\overrightarrow{AG}=\frac{2}{3}\cdot\frac{1}{2}\left(\overrightarrow{AB}+\overrightarrow{AC}\right)=\frac{1}{3}\left(\overrightarrow{AB}+\overrightarrow{AC}\right)$.

1.b) $$\left.\begin{array}{l}\vec{a}+3\vec{b}=11\vec{j}\Rightarrow\vec{j}=\frac{1}{11}\left(\vec{a}+3\vec{b}\right)\\ \vec{i}=3\vec{j}-b=\frac{3}{11}\vec{a}+\frac{9}{11}\vec{b}-\vec{b}=\frac{3}{11}\vec{a}-\frac{2}{11}\vec{b}\end{array}\right\}\Rightarrow\vec{v}=2\vec{i}-4\vec{j}=\frac{6}{11}\vec{a}-\frac{4}{11}\vec{b}-\frac{4}{11}\vec{a}-\frac{12}{11}\vec{b}=\frac{2}{11}\vec{a}-\frac{16}{11}\vec{b}$$

2.a) $$\left.\begin{array}{l}AB^2=9+1=10\\ AC^2=1+49=50\\ BC^2=4+36=40\end{array}\right\}AB^2+BC^2=AC^2\Rightarrow\triangle ABC\text{ este dreptunghic în }B.$$

2.b) $\frac{BC}{\sin A}=\frac{AB}{\sin C}\Rightarrow\sin C=\frac{AB\cdot\sin A}{BC}=\frac{4\cdot\frac{\sqrt{3}}{2}}{2\sqrt{3}}=1\Rightarrow m(\sphericalangle C)=90^\circ$.

$AC^2=AB^2-BC^2=16-12=4\Rightarrow AC=2$.

3.a) $m_{BC}=3\Rightarrow AD:\frac{y+2}{x-1}=-\frac{1}{3}\Leftrightarrow AD:x+3y+5=0$.

3.b) Ecuația dreptei AB este: $4x+\frac{4}{3}y=\frac{16}{3}$. $C\left(1;1\right)$ verifică ecuația dreptei AB, $4+\frac{4}{3}=\frac{16}{3}$.