Soluție:

1. $m \in (-\infty, 3)$.

2. **a)** $V(-1,1)$.

b) $x^2 + 2x - 2 = 0 \Rightarrow S = x_1 + x_2 = -2,\ P = x_1 \cdot x_2 = -2$. Expresia este echivalentă cu:

$$\frac{x_1^2 + 2x_1 - 2 + 3}{x_2} + \frac{x_2^2 + 2x_2 - 2 + 3}{x_1} = \frac{3}{x_2} + \frac{3}{x_1} = \frac{3S}{P} = 3.$$

3. Sistemul $\begin{cases} y = 2x - 3 \\ y = x^2 + mx + 1 \end{cases}$ nu are soluții $\Leftrightarrow$

$\Leftrightarrow x^2 + (m-2)x + 4 \neq 0, \forall x \in \mathrm{R} \Leftrightarrow \Delta = (m-2)^2 - 16 < 0 \Leftrightarrow m \in (-2, 6)$.

4. **a)** $\begin{cases} a > 0, a \neq 1 \\ \log_a 9 = -2 \end{cases} \Leftrightarrow a^{-2} = 9 \Leftrightarrow a = \frac{1}{3}$.

b) $2^{2x-1} = 3^{2x-1} \Leftrightarrow \left(\frac{2}{3}\right)^{2x-1} = 1 \Leftrightarrow 2x - 1 = 0 \Leftrightarrow x = \frac{1}{2}$.