Soluții

1. a) $f_n'(x)=n\cdot\sin^{n-1}x\cdot\cos x$ .Utilizand regula de derivare a unui produs ,se obtine relatia ceruta.

b)

$f_n''(x)=n\cdot\sin^{n-2}x\cdot\left(n-1-n\sin^2 x\right)$; $f_n''(x_n)=0, x_n\in\left(0,\frac{\pi}{2}\right)\Rightarrow n.\sin^2 x=n-1\Rightarrow \sin x_n=\sqrt{\frac{n-1}{n}}$.

c) $L=\lim\limits_{n\to\infty}(\sin x_n)^n=\lim\limits_{n\to\infty}(1+\sin x_n-1)^{\frac{1}{\sin x_n-1}\cdot n\cdot(\sin x_n-1)}$; $L=e^{\lim\limits_{n\to\infty}\left(\sqrt{\frac{n-1}{n}}-1\right)\cdot n}=e^{-\frac{1}{2}}$.

2.a) Se motiveaza ca F este derivabila pe $\mathbb{R}$ $F'(x)=\frac{(2x+a)\left(x^2+1\right)-\left(x^3+ax^2+5x\right)}{\left(x^2+1\right)\sqrt{x^2+1}}\Rightarrow$

$F'(x)=f(x),\forall x\in\mathbb{R}$,deci are loc concluzia

. b) Aria $=\int_1^2 f(x)dx=\frac{x^2+2x+5}{\sqrt{x^2+1}}\Big|_1^2=\frac{13}{\sqrt{5}}-\frac{8}{\sqrt{2}}$.

c) Cu schimbarea $t=-x$,a doua integrala devine $\int_{-2}^0 F(x)dx=\int_0^2 F(-t)dt$.

Relatia din ipoteza devine

$$\int_0^2 F(x)-F(-x)dx=2\Leftrightarrow\int_0^2\frac{2ax}{\sqrt{x^2+1}}dx=2\Leftrightarrow a\cdot\sqrt{x^2+1}\Big|_0^2=1\Rightarrow a=\frac{\sqrt{5}+1}{4}.$$