**Soluție**

**1.a)** $f'(x)=e^x-1$ .

**b)** $f''(x)=e^x \Rightarrow \lim\limits_{x\to\infty}\frac{e^x-1}{e^x}=1$ .

**c)** $e^1\geq 1+1, e^2\geq 2+1,\ldots, e^n\geq n+1 \Rightarrow e+e^2+\ldots+e^n\geq 1+2+\ldots+n+n$ ;

$\frac{e\left(e^n-1\right)}{e-1}\geq\frac{n(n+1)}{2}+n \Leftrightarrow \frac{e^{n+1}-1}{e-1}\geq\frac{n(n+3)}{2}$ c.c.t.d.

**2.a)** $\int_0^2 (x+1)\,f(x)\,dx=\int_0^2 x^3\,dx=4.$

**b)** $\int_0^1 g(x)\,dx=\int_0^1 f''(x)\,dx=f'(x)\Big|_0^1$ ; $f'(x)=\frac{2x^3+3x^2}{(x+1)^2} \Rightarrow \int_0^1 g(x)\,dx=\frac{5}{4}$ .

**c)** $\int g(x)\,dx=\int f''(x)\,dx=f'(x)+c=2x-1+\frac{1}{(x+1)^2}+c.$ O primitivă este de forma:

$G:[0,\infty)\to\mathbb{R}$ , $G(x)=2x-1+\frac{1}{(x+1)^2}+c$ ; $\lim\limits_{x\to+\infty}G(x)=\infty$ ; $\lim\limits_{x\to+\infty}\frac{G(x)}{x}=2$ ;

$\lim\limits_{x\to+\infty}\left(G(x)-2x\right)=-1+c \Rightarrow -1+c=0 \Rightarrow c=1 \Rightarrow G(x)=2x+\frac{1}{(x+1)^2}$ .