Soluție

1.a. Avem $\det(B)=\begin{vmatrix}2 & 2 & -3\\ 1 & 3 & -3\\ 1 & 2 & -2\end{vmatrix}=1.$

b. Calculând avem $A^2=\begin{pmatrix}1 & 2 & -3\\ 1 & 2 & -3\\ 1 & 2 & -3\end{pmatrix}\cdot\begin{pmatrix}1 & 2 & -3\\ 1 & 2 & -3\\ 1 & 2 & -3\end{pmatrix}=O_3.$

c. $B=\begin{pmatrix}a & 2a & -3a\\ a & 2a & -3a\\ a & 2a & -3a\end{pmatrix}+\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}a+1 & 2a & -3a\\ a & 2a+1 & -3a\\ a & 2a & -3a+1\end{pmatrix}\Rightarrow 2B=\begin{pmatrix}2a+2 & 4a & -6a\\ 2a & 4a+2 & -6a\\ 2a & 4a & -6a+2\end{pmatrix}$ și

$B^2=\begin{pmatrix}2a+1 & 4a & -6a\\ 2a & 4a+1 & -6a\\ 2a & 4a & -6a+1\end{pmatrix}\Rightarrow 2B-B^2=I_3$. Din $2B-B^2=I_3\Rightarrow B\left(2I_3-B\right)=I_3\Rightarrow B^{-1}=2I_3-B.$

2.a. $x\circ y=3xy+3x+3y+3+2-3=3x(y+1)+3(y+1)-1=(y+1)(3x+3)-1=3(x+1)(y+1)-1$.

b. $\left(x^2-5\right)\circ\left(y^2-10\right)=-1\Leftrightarrow 3(x^2-4)(y^2-9)=0\Rightarrow$ solutiile : $x\in\{-2;2\}$ sau $y\in\{-3;3\}$

c. Luăm $x=\frac{1}{3}, y=\frac{5}{2}\in\mathbb{Q}$, avem $\frac{1}{3}\circ\frac{5}{2}=3\left(\frac{1}{3}+1\right)\left(\frac{5}{2}+1\right)-1=3\cdot\frac{4}{3}\cdot\frac{7}{2}-1=14-1=13\in\mathbb{N}$. Se pot lua și alte exemple.