Solutie

1.a) **Formula**

$f(1)=0 \qquad f'(x)=e^{-\frac{1}{x}}\frac{x^2+x-1}{x^2} \qquad f'(1)=\frac{1}{e}$

$y=\frac{1}{e}(x-1)$

b) $f'(x)=0 \qquad x_{1,2}=\frac{-1\pm\sqrt{5}}{2}$

$x=\frac{-1-\sqrt{5}}{2}$ **punct de maxim local**

$x=\frac{-1+\sqrt{5}}{2}$ **punct de minim local**

c) $m=\lim\frac{f(x)}{x}=1$

$n=\lim\limits_{x\to\infty}(f(x)-x)=\lim\limits_{x\to\infty}x(e^{-\frac{1}{x}}-1)-e^{-\frac{1}{x}}=-2$

$y=x-2$ **asimptota oblica spre** $+\infty$

2.a) $f_1(1)=\int_0^1 t\sqrt{t^2+1}dt$

$=\frac{1}{2}\int_0^1(t^2+1)'\sqrt{t^2+1}dt$

$=\frac{1}{3}(t^2+1)\sqrt{t^2+1}\,|_0^1=\frac{2\sqrt{2}-1}{3}$

b) $f_n'(x)=x^n\sqrt{x^2+1} \qquad \forall x\in[0,+\infty)$

$f_n'(x)\geq 0 \qquad \forall x\geq 0$

f_n **strict crescatoare** $\forall x\in[0,+\infty)$

c) $L=\lim\limits_{x\to\infty}f_n(x)=\lim\limits_{x\to\infty}\int_0^x t^n\sqrt{t^2+1}dx\geq\int_0^x t^n dx=\infty$

Cazul $\frac{\infty}{\infty}$ **si aplicand L'Hopital, obtinem** $L=\lim\limits_{x\to\infty}\frac{x^n\sqrt{x^2+1}}{(n+2)x^{n+1}}$, $L=\frac{1}{n+2}$