Soluție

1.a) $\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to\infty} \frac{f(x)}{x} = +\infty$

f nu admite asimptota spre $+\infty$

b) $f'(x) = 0 \quad (n+1)x^n - (n+2) = 0$

$x_n = \sqrt[n]{\frac{n+2}{n+1}}$

c) $\lim_{n\to\infty} x_n = \lim_{n\to\infty} \sqrt[n]{\frac{n+2}{n+1}} = 1$

2.a) $I_1 = \int_0^1 \frac{x^2+1-1}{1+x^2}dx = \int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx =$

$= x \Big|_0^1 - arctg(x) \Big|_0^1 = 1 - \frac{\pi}{4}$

b) $I_{n+1} + I_n = \int_0^1 \frac{x^{2n+2}+x^{2n}}{1+x^2}dx =$

$= \int_0^1 x^{2n}dx = \frac{1}{2n+1}$

c) $\frac{x^{2n}}{1+x^2} \le x^{2n},\ (\forall) x \in [0,1] \qquad \Rightarrow 0 < I_n \le \int_0^1 x^{2n}dx = \frac{1}{2n+1} \to 0$

$\lim_{n\to\infty} I_k = 0$