

Soluții

$$\mathbf{1.a)} \quad \left. \begin{array}{l} \overrightarrow{AO} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AD}). \\ \overrightarrow{AB} = \overrightarrow{DC} \text{ și } \overrightarrow{AD} = \overrightarrow{BC} \end{array} \right\} \Rightarrow \overrightarrow{AO} = \frac{1}{2}(\overrightarrow{DC} + \overrightarrow{BC}).$$

1.b)

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$|\overrightarrow{AB} + \overrightarrow{AD}| = |\overrightarrow{AC}| = |2 \cdot \overrightarrow{OC}| \sqrt{3} = |2 \cdot \overrightarrow{DC}| \cdot \sin(\widehat{CDO}) = 2 \cdot 1 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}, \text{ unde O este intersecția diagonalelor rombului.}$$

$$\mathbf{2.a)} \quad BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A = 100 + 144 + 240 \cdot \frac{1}{2} = 364 \Rightarrow BC = 2\sqrt{91}.$$

$$\mathbf{2.b)} \quad AB: \frac{x-1}{1} = \frac{y+2}{3} \Leftrightarrow AB: 3x - y - 5 = 0. \quad C(0; -5) \text{ verifică ecuația dreptei } AB: 0 + 5 - 5 = 0.$$

$$\mathbf{3.a)} \quad AB: \frac{x-1}{1} = \frac{y+1}{4}.$$

$$\text{Înlocuind } x = \frac{1}{2} \Rightarrow y = -3$$

$$\mathbf{3.b)} \quad AC = \sqrt{BC^2 - AB^2} = \sqrt{12} = 2\sqrt{3} \Rightarrow AM = \frac{AC}{2} = \sqrt{3}$$

$$BM = \sqrt{AM^2 + AB^2} = \sqrt{3+4} = \sqrt{7}.$$