

Soluții

$$\mathbf{1.a)} \quad \left. \begin{array}{l} \overrightarrow{OA} = \overrightarrow{OD} + \overrightarrow{DA} \\ \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} \end{array} \right\} \Rightarrow \overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{OB} \text{ căci } \overrightarrow{DA} + \overrightarrow{BC} = \mathbf{0}$$

1.b)

$$\overrightarrow{AG} = \frac{2}{3} \overrightarrow{AM} = \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}} (\overrightarrow{AB} + \overrightarrow{AC}) = \frac{1}{3} (\overrightarrow{AB} + \overrightarrow{AC})$$

$$\left. \begin{array}{l} \overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG} = \overrightarrow{OA} + \frac{1}{3} (\overrightarrow{AB} + \overrightarrow{AC}) \\ \overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG} = \overrightarrow{OB} + \frac{1}{3} (\overrightarrow{BA} + \overrightarrow{BC}) \\ \overrightarrow{OG} = \overrightarrow{OC} + \overrightarrow{CG} = \overrightarrow{OC} + \frac{1}{3} (\overrightarrow{CA} + \overrightarrow{CB}) \end{array} \right\} \Rightarrow 3\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \frac{1}{3} \underbrace{(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{CB})}_{\mathbf{0}} \Rightarrow \mathbf{2.}$$

$$\overrightarrow{OG} = \frac{1}{3} (\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$$

$$2AB^2 = 27^2 \Rightarrow AB = \frac{27\sqrt{2}}{2}$$

a)

$$\sigma[ABC] = \frac{27^2 \cdot \cancel{2}}{\cancel{2} \cdot 4} = \left(\frac{27}{2} \right)^2 = \frac{729}{4}$$

$$\mathbf{2.b)} \quad BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A = 100 + 400 - 400 \cdot \frac{1}{2} = 500 - 200 = 300 \Rightarrow \\ \Rightarrow BC = 10\sqrt{3}$$

$$\mathbf{3.a)} \quad AB: \frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A} \Leftrightarrow AB: \frac{y+3}{x} = \frac{3}{1} \Rightarrow y - 3x + 3 = 0 \Rightarrow \\ AB: 3x - y - 3 = 0$$

3.b) Fie $OE \perp AB$, $E \in [AB]$

$$OE = \frac{OB \cdot OA}{AB} = \frac{3 \cdot 1}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$