Soluții

1.a) $\left.\begin{array}{l}\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} \\ \overrightarrow{AB} = \overrightarrow{DC} = 2\overrightarrow{MC}\end{array}\right\} \Rightarrow \overrightarrow{AM} = 2\overrightarrow{MC} + \overrightarrow{BM}.$

1.b) $\left.\begin{array}{l}3\overrightarrow{OA} \text{ are coordonatele } (3;6). \\ -\overrightarrow{OB} \text{ are coordonatele } (1;-3). \\ \overrightarrow{OC} \text{ are coordonatele } (1;0).\end{array}\right\} 3\overrightarrow{OA} - \overrightarrow{OB} + \overrightarrow{OC}$ are coordonatele $(5;3)$.

2.a) $\triangle ABC$ este isoscel deoarece $m(\sphericalangle ACB) = 180^\circ - 120^\circ - 30^\circ = 30^\circ = m(\sphericalangle ABC)$.

Fie $AD \perp BC,\ D \in [BC]$. $\cos(\widehat{BAD}) = \dfrac{\sqrt{3}}{2} = \dfrac{AD}{AB} = \dfrac{\frac{AC}{2}}{AB} \Rightarrow AC = 20\sqrt{3}$.

$BD = \sin(\widehat{BAD}) \cdot AB = 10$.

$\sigma[ABC] = \dfrac{BD \cdot AC}{2} = 100\sqrt{3}$.

2.b) $BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A \Rightarrow BC = \sqrt{400 + 1600 - 800} = 20\sqrt{3}$.

Observăm că $AB^2 + BC^2 = BC^2\,(400 + 1200 = 1600) \Rightarrow \triangle ABC$ este dreptunghic în B.

$\sigma[ABC] = \dfrac{AB \cdot BC}{2} = 200\sqrt{3}$.

3.a) $M(2;3)$, M mijlocul segmentului $[AB]$.

$m_{AB} = 1 \Rightarrow AM : \dfrac{y-3}{x-2} = -1 \Leftrightarrow AM : x + y - 5 = 0$.

3.b) $M(0;3), M$ mijlocul segmentului $[BC]$.

$AM : y = 3$.