Soluție

a) $16x^2 \geq (x^2+4)^2$, pentru orice $x \in (0,+\infty)$; $(x^2-4)^2 \geq 0$, pentru orice $x \in (0,+\infty)$.

b) f continuă, $f \geq 0$ $pe\,[0,1]$; $aria(\Gamma_f) = \int_0^1 (x^2+1)dx = (\frac{1}{3}x^3 + x)\Big|_0^1 = \frac{4}{3}$.

c) $(\frac{1}{2}x^2 - 2x)\Big|_0^n \leq (\frac{n}{2}x^2 - 4x)\Big|_0^2$; $n = 4$.

d) $f:(0,+\infty) \to \mathbb{R}, f(x) = \frac{1}{\sqrt{x}} - \frac{1}{x}$; $F:\mathbb{R} \to \mathbb{R}, F(x) = 2\sqrt{x} - \ln x + C, C \in \mathbb{R}$; $F(1) = 1$ rezultă $C = -1$;
$F(x) = 2\sqrt{x} - \ln x - 1$.

e) $f, g:[1,e] \to \mathbb{R}, f(x) = \frac{x}{x^2+4}, g(x) = \frac{x^2+4}{16x}$; continue pe $[1,e]$, $f \leq g$; $\int_1^e \frac{x}{x^2+4}dx \leq \int_1^e \frac{x^2+4}{16x}dx = \frac{e^2+7}{32}$.

f) $I = -\int_1^e (\ln x - 1)dx + \int_e^{e^2} (\ln x - 1)dx = 2(e-1)$.