

Soluții

$$\mathbf{1.a)} \quad \left. \begin{aligned} \overrightarrow{MO} &= \frac{1}{2}(\overrightarrow{MA} + \overrightarrow{MC}) \\ \overrightarrow{MO} &= \frac{1}{2}(\overrightarrow{MB} + \overrightarrow{MD}) \end{aligned} \right\} \Rightarrow 4\overrightarrow{MO} = (\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} + \overrightarrow{MD})$$

b)

$$\overrightarrow{GA} = \frac{2}{3}\overrightarrow{MA} = -\frac{2}{3}\overrightarrow{AM} = -\frac{1}{3}(\overrightarrow{AB} + \overrightarrow{AC}).$$

$$\text{Analog obținem } \overrightarrow{GB} = -\frac{1}{3}(\overrightarrow{BA} + \overrightarrow{BC}) \text{ și } \overrightarrow{GC} = -\frac{1}{3}(\overrightarrow{CB} + \overrightarrow{CA}).$$

$$\text{Deci } \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = -\frac{1}{3}(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CB} + \overrightarrow{CA}) = -\frac{1}{3} \left(\overbrace{\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}}^{\vec{0}} + \overbrace{\overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BA}}^{\vec{0}} \right) = \vec{0}$$

Cum $m(\sphericalangle A) = 30^\circ$ și $m(\sphericalangle B) = 120^\circ \Rightarrow m(\sphericalangle C) = 30^\circ \Rightarrow \triangle ABC$ este isoscel.

$$BD \perp AC, \text{ unde D este mijlocul segmentului } [AC]. \sin(\widehat{BCD}) = \frac{BD}{BC} = \frac{1}{2} \Rightarrow BD = 5.$$

$$\mathbf{2.a)} \quad \cos(\widehat{BCD}) = \frac{CD}{BC} \Rightarrow CD = 5\sqrt{3} = \frac{1}{2} \cdot AC \Rightarrow AC = 10\sqrt{3}$$

$$\sigma[ABC] = \frac{BD \cdot AC}{2} = \frac{5 \cdot 10\sqrt{3}}{2} = 25\sqrt{3}.$$

$$\mathbf{2.b)} \quad BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A \Rightarrow BC = 3\sqrt{10 + 2\sqrt{6}}$$

$$\mathbf{3.a)} \quad AC^2 + BC^2 = AB^2 \Leftrightarrow 2 + (k-1)^2 + 4 + (k+1)^2 = 16 + 4 \Leftrightarrow 2k^2 = 10 \Rightarrow k = \sqrt{5}.$$

$$\mathbf{3.b)} \quad m_{BC} = \frac{\frac{11}{3}}{4} = \frac{11}{12} \Rightarrow \frac{y-1}{x+\frac{1}{2}} = -\frac{12}{11} \Rightarrow d: 12x + 11y - 5 = 0.$$