Soluție

1.a $f'(x)=\dfrac{3x^2+6x+2}{3\sqrt[3]{x^3+3x^2+2x+1}}-\dfrac{3x^2-1}{\sqrt[3]{x^3-x+1}}$

$f(0)=0,\ \mathrm{f}'(0)=3$

y=3x

b $\lim\limits_{x\to\infty} f(x)=1$

$y=1$ asimptota orizontala spre $+\infty$

c $f(k)=\sqrt[3]{k(k+1)(k+2)+1}-\sqrt[3]{(k-1)k(k+1)+1}$

$\sum\limits_{k=1}^{n} f(k)=\sqrt[3]{n^3+3n^2+2n+1}-1$

$l=e^{\lim\limits_{n\to\infty}(\sqrt[3]{n^3+3n^2+2n+1}-n-1)}=e^{-\frac{1}{3}}$

2.a $f_1(e)=\int_{\frac{1}{e}}^{e}\left(\dfrac{t^2}{2}\right)'\ln t dt=\dfrac{t^2\ln t}{2}-\dfrac{1}{2}\int_{\frac{1}{e}}^{e} t dt$

$f_1(e)=\dfrac{e^4}{4}+\dfrac{3}{4e^2}$

b $f'(x)=x^n\ln x$

$x\in(0;1)\Rightarrow \ln x\le 0\Rightarrow f'(x)\le 0$

$f\ descrescatoare\ pe\ (0;1)$

$-1\le \ln t\le 0\ \forall \mathrm{x}\in\left[\dfrac{1}{\mathrm{e}};1\right]$

c $-\int_{\frac{1}{\mathrm{e}}}^{1} t^n\le f_n(1)\le 0$

$-\dfrac{1}{n+1}+\dfrac{1}{(n+1)e^{n+1}}\le f_n(1)\le 0$

$\lim\limits_{n\to\infty} f_n(1)=0$