Rezolvare

1) **a)** $\lim\limits_{x\to 0}\dfrac{f(x)}{x^2-x}=\lim\limits_{x\to 0}\dfrac{\arcsin x}{x}=1.$

b) $x\in(-1,1)\to f'(x)=\arcsin x+\dfrac{x-1}{\sqrt{1-x^2}}=\arcsin x+\dfrac{x-1}{\sqrt{(1-x)(1+x)}}.$

$\lim\limits_{x\nearrow 1} f'(x)=\dfrac{\pi}{2}\Rightarrow f'(1)=\dfrac{\pi}{2}.$ f nu este derivabilă în -1.

c) $f''(x)=\dfrac{1}{\sqrt{1-x^2}}-\dfrac{\sqrt{1-x^2}-(x-1)\cdot\dfrac{(-x)}{\sqrt{1-x^2}}}{1-x^2}=\dfrac{1}{\sqrt{1-x^2}}+\dfrac{1-x}{(1-x^2)\cdot\sqrt{1-x^2}}\geq 0\Rightarrow f$ este convexă.

2) **a)** $\left.\begin{array}{l}\mathrm{F}'(x)=f(x)=\dfrac{x^5-1}{x-1},\ \forall x\neq 1\\ \mathrm{F}'(1)=f(1)=5\end{array}\right\}\Rightarrow \mathrm{F}'(x)>0,\ \forall x\in\mathbb{R}\Rightarrow \mathrm{F}$ este strict crescătoare pe $\mathbb{R}$.

b) $\mathrm{F}(x)=\dfrac{x^5}{5}+\dfrac{x^4}{4}+\dfrac{x^3}{3}+\dfrac{x^2}{2}+x$ deci $\lim\limits_{x\to-\infty}\mathrm{F}(x)=-\infty$ şi $\lim\limits_{x\to+\infty}\mathrm{F}(x)=+\infty$. F fiind continuă, rezultă că F este surjectivă, deci conform lui (**a**) este bijectivă.

c) $\int_0^a F'(x)\,dx=\int_0^1 t\cdot f(t)\,dt=\left(\dfrac{t^2}{2}+\dfrac{t^3}{3}+\dfrac{t^4}{4}+\dfrac{t^5}{5}+\dfrac{t^6}{6}\right)\Big|_0^1=\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}+\dfrac{1}{6}.$