

**Soluții**

1. a)  $A^2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}; 2I_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$

b)  $A - xI_2 = \begin{pmatrix} 1-x & 1 \\ 1 & -1-x \end{pmatrix}; \det(A - xI_2) = x^2 - 2; x^2 - 2 = 0 \Leftrightarrow x = \pm\sqrt{2}.$

c)  $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}, AX = \begin{pmatrix} x+z & y+t \\ x-z & y-t \end{pmatrix}; XA = \begin{pmatrix} x+y & x-y \\ z+t & z-t \end{pmatrix} \Rightarrow y = z, t = x - 2y \Rightarrow X = \begin{pmatrix} x & y \\ y & x - 2y \end{pmatrix},$   
 $x, y \in \mathbb{R}.$

2. a)  $3, 2 \in \mathbb{Z}$  și  $3^2 - 2 \cdot 2^2 = 1$ , deci  $3 + 2\sqrt{2} \in G.$

b)  $x = a_1 + b_1\sqrt{2}, y = a_2 + b_2\sqrt{2}, a_1, b_1, a_2, b_2 \in \mathbb{Z}$  și  $a_1^2 - 2b_1^2 = 1, a_2^2 - 2b_2^2 = 1;$   
 $xy = (a_1 + b_1\sqrt{2})(a_2 + b_2\sqrt{2}) = (a_1a_2 + 2b_1b_2) + (a_1b_2 + a_2b_1)\sqrt{2}; a_1a_2 + 2b_1b_2, a_1b_2 + a_2b_1 \in \mathbb{Z};$   
 $(a_1a_2 + 2b_1b_2)^2 - 2(a_1b_2 + a_2b_1)^2 = (a_1^2 - 2b_1^2)(a_2^2 - 2b_2^2) = 1.$

c)  $x = a + b\sqrt{2}, a^2 - 2b^2 = 1 \Rightarrow x \neq 0; x^{-1} = a - b\sqrt{2}, a^2 - 2(-b)^2 = 1 \Rightarrow x^{-1} \in G.$