Rezolvare

1) **a)** $f_n(x)=\dfrac{x^{2n+1}+1}{x+1},\ \forall x\neq -1 \Rightarrow f_n'(x)=\dfrac{g_n'(x)}{x+1}-\dfrac{g_n(x)}{(x+1)^2}.$

b) $\lim\limits_{n\to\infty} f_n'\left(\dfrac{1}{2}\right)=\lim\limits_{n\to\infty}\left[\dfrac{(2n+1)\cdot\left(\dfrac{1}{2}\right)^{2n}}{\dfrac{3}{2}}-\dfrac{\left(\dfrac{1}{2}\right)^{2n+1}+1}{\dfrac{9}{4}}\right]=-\dfrac{4}{9}.$

c)

2) **a)** $\mathrm{I}_2=\int_0^2\left(2x-x^2\right)^2dx=\int_0^2\left(4x^2-4x^3+x^4\right)dx=\left(\dfrac{4x^3}{3}-\dfrac{4x^4}{4}+\dfrac{x^5}{5}\right)\Bigg|_0^2=\dfrac{32}{3}-16+\dfrac{32}{5}.$

b) $\mathrm{I}_n=\int_0^2\left(2x-x^2\right)^n\cdot(x)'\,dx=\left(2x-x^2\right)^n\cdot x\Big|_0^2-n\int_0^2\left(2x-x^2\right)^{n-1}\cdot(2-2x)\cdot x=$

$=-2n\int_0^2\left(2x-x^2\right)^{n-1}\cdot\left(-x^2+x\right)dx=-2n\int_0^2\left(2x-x^2\right)^{n-1}\left(-x^2+2x-x\right)dx=$

$=-2n\int_0^2\left(2x-1^2\right)^n dx+2n\int_0^2 x\cdot\left(2x-x^2\right)^{n-1}\Rightarrow(2n+1)\mathrm{I}_n=2n\mathrm{I}_{n-1}.$

c) Conform punctului (**b**) $\mathrm{I}_n=\dfrac{(2n)!!}{(2n+1)!!}\Rightarrow\lim\limits_{n\to\infty}\mathrm{I}_n=0.$