

Soluții

$$\left. \begin{array}{l} \overrightarrow{GM} = \frac{1}{2}(\overrightarrow{GB} + \overrightarrow{GC}) \\ \text{1.a) } \overrightarrow{GN} = \frac{1}{2}(\overrightarrow{GC} + \overrightarrow{GA}) \\ \overrightarrow{GP} = \frac{1}{2}(\overrightarrow{GA} + \overrightarrow{GB}) \end{array} \right\} \Rightarrow \overrightarrow{GM} + \overrightarrow{GN} + \overrightarrow{GP} = \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = -(\overrightarrow{AG} + \overrightarrow{BG} + \overrightarrow{CG}) = \vec{0}$$

1.b) Vectorul $2\vec{u} - 3\vec{v}$ are coordonatele $(-4 + 3; 4 - 9) = (-1; -5)$.

Se verifică egalitatea: $BC^2 + AC^2 = AB^2 \Rightarrow \triangle ABC \Leftrightarrow 25 + 144 = 169 \Rightarrow \triangle ABC$ este dreptunghic în C .

$$\text{2.a) } \sigma[ABC] = \frac{BC \cdot AC}{2} = \frac{60}{2} = 30.$$

$$\text{2.b) } \frac{BC}{\sin A} = \frac{AC}{\sin B} \Leftrightarrow \frac{3\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{6}{\sin B} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow m(\sphericalangle B) = 45^\circ.$$

3.a) $M(1;1)$, M este mijlocul segmentului $[BC]$.

$$AM = \sqrt{(x_M - x_A)^2 + (y_M - y_A)^2} = \sqrt{4 + 0} = 2.$$

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B} = -\frac{1}{7}. \text{ Fie } D \in BC, AD \perp BC.$$

$$\text{3.b) } m_{AD} = -\frac{1}{m_{BC}} = 7 \Leftrightarrow \frac{y+2}{x-5} = 7 \Rightarrow AD: 7x - y - 37 = 0$$