

Soluție

1. a) $2 \cdot \overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AB} + \overrightarrow{AD} = 2 \cdot \overrightarrow{AB} + \overrightarrow{AD}.$

b) $\overrightarrow{MB} + \overrightarrow{MD} = 2 \cdot \overrightarrow{MC}, \overrightarrow{MA} + \overrightarrow{ME} = 2 \cdot \overrightarrow{MC},$

$$\overrightarrow{MA} + \overrightarrow{ME} + \overrightarrow{MC} + \overrightarrow{MB} + \overrightarrow{MD} = \overrightarrow{MA} + \overrightarrow{ME} + \frac{1}{2} \cdot (\overrightarrow{MA} + \overrightarrow{ME}) + 2 \cdot \frac{1}{2} \cdot (\overrightarrow{MA} + \overrightarrow{ME}).$$

2. a) Se notează cu x lungimea catetei, perimetrul este $2x + x\sqrt{2} = 8 + 4\sqrt{2}$, deci cateta este de lungime $x = 4$.

Aria este $\frac{x^2}{2} = 8$.

b) $\frac{BC}{\sin A} = \frac{AC}{\sin B}, \sin A = \frac{\sqrt{5}}{5}.$

3. a) $AB^2 + AC^2 = BC^2$, deci $81 + 9 + 4 + (y + 2)^2 = 121 + (y - 1)^2$. Se obține $y = 4$.

b) $AB = 3\sqrt{10}, AC = 2\sqrt{10}, BC = \sqrt{130}.$