

Soluție

1.a. $A^2 = A \cdot A = 2^2 \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} \Rightarrow \det A^2 = 4(25 - 16) = 36.$

b. $A = 2 \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}; A^2 = A \cdot A = 2^2 \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}; A^3 = A^2 \cdot A = 2^3 \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}.$

c. $A^2 - 8A + 4I_2 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix} - \begin{pmatrix} 24 & 16 \\ 16 & 24 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0_2.$

2.a. Se verifică ușor prin calcul că $\hat{b}^3 = \hat{b}$. $\hat{0}^3 = \hat{0}; \hat{1}^3 = \hat{1}; \hat{2}^3 = \hat{2}; \hat{3}^3 = \hat{3}; \hat{4}^3 = \hat{4}; \hat{5}^3 = \hat{5}.$

b. $f(\hat{2}) = \hat{0} \Leftrightarrow \hat{5}a = \hat{4} \Leftrightarrow \hat{a} = \hat{2}.$

c. Pentru $a = \hat{2}$ avem $f = X^3 + \hat{5}X = x(x - \hat{1})(x - \hat{5}).$