Soluție

a) $1+\frac{1}{x-1}-\frac{1}{x}=\frac{x(x-1)+x-(x-1)}{x(x-1)}=\frac{x^2-x+1}{x^2-x}=f(x), x\in\mathbb{R}-\{0,1\}$.

b) $\lim\limits_{x\to\pm\infty} f(x)=1\Rightarrow y=1$ asimptotă orizontală spre $\pm\infty$

$\Rightarrow$ graficul funcției nu are asimptotă oblică

$$\left.\begin{array}{l} l_s(0)=\lim\limits_{x\nearrow 0} f(x)=+\infty \\ l_d(0)=\lim\limits_{x\searrow 0} f(x)=-\infty \end{array}\right\}\Rightarrow x=0 \text{ asimptotă verticală}$$

$$\left.\begin{array}{l} l_s(1)=\lim\limits_{x\nearrow 0} f(x)=-\infty \\ l_d(1)=\lim\limits_{x\searrow 0} f(x)=+\infty \end{array}\right\}\Rightarrow x=1 \text{ asimptotă verticală}$$

c) $f'(x)=-\frac{1}{(x-1)^2}+\frac{1}{x^2}=\frac{-2x+1}{x^2(x-1)^2}, x\in\mathbb{R}-\{0,1\}$.

d) $y-f(2)=f'(2)(x-2)$

$y-\frac{3}{2}=-\frac{3}{4}(x-2)\Rightarrow 4y+3x-12=0$

e) $\lim\limits_{x\to\infty} x\left[f(x+1)+f(x+2)+..........+f(x+2008)\right]=$

$\lim\limits_{x\to\infty} x\left[1+\frac{1}{x}-\frac{1}{x+1}+1+\frac{1}{x+1}-\frac{1}{x+2}+.........+1+\frac{1}{x+2007}-\frac{1}{x+2008}\right]=$

$\lim\limits_{x\to\infty} x\left(2008+\frac{1}{x}-\frac{1}{x+2008}\right)=+\infty$

f) f continuă în $x=1$ $\Rightarrow l_s(1)=l_d(1)=g(1)$ $\Rightarrow 2a-5=a^2-13$

$a^2-2a-8=0$

$a_{1,2}=\frac{2\pm\sqrt{36}}{2}\Rightarrow a_1=4, a_2=-2$.