Rezolvare

1.a. Din f continuă în x=0 $\Rightarrow$ $l_s(0) = l_d(0) = f(0)$ și $\lim\limits_{\substack{x\to 0\\ x<0}} f(x) = \lim\limits_{\substack{x\to 0\\ x<0}} \left(e^x - 1\right) = 0$,

$\lim\limits_{\substack{x\to 0\\ x>0}} \left(x^2 + x + a\right) = a$, $f(0) = a$ obținem $a = 0$.

b. Din $f'(x) = e^x, f'(-1) = e^{-1} \Rightarrow$ $\text{y-}\frac{1}{\text{e}} + 1 = \frac{1}{e}(x+1) \Rightarrow \text{x} - \text{ye+2} - \text{e=0}$

c. $\lim\limits_{x\to -\infty} \frac{f(x)+1}{x^2 + x} = \lim\limits_{x\to -\infty} \frac{e^x}{x^2 + x} = \lim\limits_{x\to \infty} \frac{1}{e^x\left(x^2 - x\right)} = 0$

2.a. $I_0 = \int\limits_2^3 \frac{1}{x^2 - 1} dx = \frac{1}{2} \ln \frac{x-1}{x+1} \Big|_2^3 = \frac{1}{2} \ln \frac{3}{2}$.

b. $I_1 = \int\limits_2^3 \frac{x}{x^2 - 1} dx = \frac{1}{2} \ln\left(x^2 - 1\right) \Big|_2^3 = \frac{1}{2} \ln \frac{8}{3}$.

c. $I_{n+2} - I_n == \int\limits_2^3 \frac{x^n (x^2 - 1)}{x^2 - 1} dx = \frac{x^{n+1}}{n+1} \Big|_2^3 = \frac{3^{n+1} - 2^{n+1}}{n+1}$.