**Solutie**

**1.a** $f'(x) = \dfrac{1}{x\ln x}$ $\quad f'(e) = \dfrac{1}{e}$

$f(e) = 0$ , $\quad y = \dfrac{1}{e}(x-e)$

**b)** $f''(x) = -\dfrac{\ln x + 1}{(x\ln x)^2} < 0, \forall x > 1$,deci f este concave

**c)** Conform teoremei lui Lagrange exista $c_x \in (x, x+1)$ $\quad a.i.\ f(x+1) - f(x) = f'(c_x) = \dfrac{1}{c_x \ln c_x}$

avem de calculat $\lim\limits_{x\to\infty} \dfrac{x\ln x}{c_x \ln c_x}$ dar cum $\dfrac{x\ln x}{(x+1)\ln(x+1)} < \dfrac{x\ln x}{c_x \ln c_x} < 1$ limita cautata este 1.

**2a)** $\displaystyle\int_0^{\frac{\pi}{2}} f(x)\,dx = arctg(\sin x)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

**b)** Fie F o primitiva a lui f

$\Rightarrow F'(x) = \dfrac{\cos x}{1+\sin^2 x} \geq 0, (\forall) x \in [0, \dfrac{\pi}{2}]$

$\Rightarrow$ F strict crescatoare pe $[0, \infty)$

**c**) cu substitutia $\quad x = 2\pi - y \;\; obtinem \;\; I = \displaystyle\int_0^{2\pi} (2\pi - y) f(y)\,dy = 2\pi \int_0^{2\pi} f(y)\,dy - I$

de unde I=0.