

Soluție:

1. $f(0) + f(1) + f(2) + f(3) = -3 - 1 + 2 + 1 = -1.$

2. a) $\Delta = m^2 - 4m + 16 > 0, \forall m \in \mathbb{R} \Rightarrow x_1, x_2 \in \mathbb{R}, x_1 \neq x_2.$

$$G_f \cap Ox = \{A(x_1, 0), B(x_2, 0)\}; d(A, B) = |x_2 - x_1| \geq 4. \text{ Deci: } |x_2 - x_1| \geq 4 \Leftrightarrow (x_2 - x_1)^2 \geq 16 \Leftrightarrow \\ \Leftrightarrow S^2 - 4P \geq 16 \Leftrightarrow m^2 - 4m + 16 \geq 16 \Leftrightarrow m^2 - 4m \geq 0 \Rightarrow m \in (-\infty, 0] \cup [4, \infty).$$

b) $f(x) = x^2 - 4x$. Axa de simetrie este dreapta de ecuație: $x = -\frac{b}{2a} \Rightarrow x = 2.$

3. $V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \Rightarrow V\left(\frac{a+1}{2}, -\frac{(a+1)^2 - 8}{4}\right) \in d: y = x \Leftrightarrow \frac{a+1}{2} = -\frac{(a+1)^2 - 8}{4} \Leftrightarrow a \in \{-5; 1\}.$

4. a) Condiție de existență: $x > 0; \log_2 6x = \log_2 (3x + 6) \Leftrightarrow 6x = 3x + 6 \Leftrightarrow x = 2.$

b) $2^{\sqrt{x^2+2}} = 2^{1-x} \Leftrightarrow \sqrt{x^2+2} = 1-x \wedge 1-x \geq 0 \Rightarrow x^2 + 2 = 1 - 2x + x^2 \Rightarrow x = -\frac{1}{2}.$