Soluții

1.a) $\overrightarrow{DE}=\frac{1}{2}\overrightarrow{BC}\Rightarrow\overrightarrow{BC}=2\cdot\overrightarrow{DE}$

1.b) $\left.\begin{array}{l}3\vec{a}=3\vec{i}+3\vec{j}\\-2\vec{b}=2\vec{i}-4\vec{j}\end{array}\right\}\Rightarrow 3\vec{a}-2\vec{b}=5\vec{i}-\vec{j}=\vec{v}$

2.a) $\cos C=\frac{AC^2+BC^2-AB^2}{2\cdot AC\cdot BC}=\frac{\frac{2}{4}+\frac{3}{4}-\frac{6+2+4\sqrt{3}}{16}}{\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\sqrt{2}}{2}}=\frac{20-8-4\sqrt{3}}{16\cdot\frac{\sqrt{3}\cdot\sqrt{2}}{2}}=\frac{12-4\sqrt{3}}{8\sqrt{6}}=\frac{3-\sqrt{3}}{2\sqrt{6}}=$

$=\frac{\sqrt{3}\left(\sqrt{3}-1\right)}{2\cdot\sqrt{3}\cdot\sqrt{2}}=\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}.$

2.b) Fie $BD\perp AC, D\in[AC]$

$\sin(\widehat{ABD})=\sin(30^\circ)=\frac{AD}{AB}\Rightarrow AD=\frac{10}{2}=5.$

$\cos(\widehat{ABD})=\cos(30^\circ)=\frac{BD}{AB}\Rightarrow BD=5\sqrt{3}$

$\sigma[ABC]=\frac{BD\cdot AC}{2}=\frac{20\cdot5\sqrt{3}}{2}=50\sqrt{3}.$

3.a) $O(0;0)$ este mijlocul segmentului $[AC]$.

$O(0;0)$ este și mijlocul segmentului $[BD]$ pentru ca $ABCD$ sa fie paralelogram

$\left.\begin{array}{l}\Rightarrow 2+x_D=0\Rightarrow x_D=-2\\\Rightarrow 2+y_D=0\Rightarrow y_D=-2\end{array}\right\}\Rightarrow D\left(-2;-2\right)$

3.b) $AB:\frac{x+1}{2}=\frac{y+7}{4}$. Coordonatele lui C verifică ecuația dreptei AB, $\frac{3+1}{2}=\frac{1+7}{4}=2.$