Soluție

1. a) $f'(x)=\left(\dfrac{x^2+3}{x-1}\right)'=\dfrac{\left(x^2+3\right)'(x-1)-(x-1)'\left(x^2+3\right)}{(x-1)^2}=\dfrac{2x(x-1)-\left(x^2+3\right)}{(x-1)^2}=\dfrac{x^2-2x-3}{(x-1)^2}, \forall x\neq 1$.

b) $f'(x)=0\Rightarrow x^2-2x-3=0\Rightarrow x\in\{-1,3\}$ .Din tabel $\Rightarrow A(-1,-2)$ maxim şi $B(3,6)$ minim pt. funcția f .

c) Folosind din nou tabelul obținem: $f(a)\leq -2, \forall a\in(-\infty,1)$ şi de asemenea $f(b)\geq 6, \forall b\in(1,+\infty)$.

Scăzând acum cele două inegalități obținem: $f(a)-f(b)\leq -8$, pentru orice $a<1$ şi $b>1$.

2. a) $\int f^2(x)\,dx=\int\left(\sqrt{2x-1}\right)^2dx=\int(2x-1)\,dx=\int 2x\,dx+\int 1\,dx=x^2-x+\mathcal{C}$.

b) $\displaystyle\int_1^5\sqrt{2x-1}\,dx \underset{u'(x)=2}{\overset{u(x)=2x-1}{=}} \frac{1}{2}\int_1^5 u'(x)\cdot u^{\frac{1}{2}}(x)\,dx=\left.\frac{u^{\frac{1}{2}+1}(x)}{\frac{1}{2}+1}\right|_1^5=\left.\frac{1}{3}u(x)\sqrt{u(x)}\right|_1^5=\frac{26}{3}$.

c) $\displaystyle\int_1^5 f(x)\cdot F^{2008}(x)\,dx=\int_1^5 F'(x)\cdot F^{2008}(x)\,dx=\left.\frac{F^{2009}(x)}{2009}\right|_1^5=\frac{F^{2009}(5)}{2009}-\frac{F^{2009}(1)}{2009}=\frac{3^{6027}-1}{2009\cdot 3^{2009}}$.