Soluție

1. a) Avem $f'(x)=\left(x^2+e^x\right)'=\left(x^2\right)'+\left(e^x\right)'=2x+e^x, \forall x\in\mathbb{R}\Rightarrow f'(0)=2\cdot 0+e^0=1$.

b) Din punctul a) avem $f''(x)=\left(f'(x)\right)'=\left(2x+e^x\right)'=2+e^x\geq 0, \forall x\in\mathbb{R}$, adică f este convexă pe $\mathbb{R}$.

c) $\lim\limits_{x\to+\infty}\dfrac{f'(x)}{e^x}\overset{\text{cf. pct. a)}}{=}\lim\limits_{x\to+\infty}\dfrac{2x+e^x}{e^x}\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}}\lim\limits_{x\to+\infty}\dfrac{\left(2x+e^x\right)'}{\left(e^x\right)'}=\lim\limits_{x\to+\infty}\dfrac{2+e^x}{e^x}=\lim\limits_{x\to+\infty}\left(\dfrac{2}{\underbrace{e^x}_{\to 0}}+1\right)=1$.

2. a) Avem $\int_0^1 f(x)dx=\int_0^1\left(e^x-x\right)dx=\int_0^1 e^x\,dx-\int_0^1 x\,dx=\left(e^x-\dfrac{x^2}{2}\right)\Bigg|_0^1=e-\dfrac{3}{2}$.

b) $\int_0^1 x\cdot f(x)dx=\int_0^1\left(xe^x-x^2\right)dx=\int_0^1 x\cdot\left(e^x\right)'dx-\left(\dfrac{x^3}{3}\right)\Bigg|_0^1=\left(x\cdot e^x\right)\Big|_0^1-\int_0^1 e^x\,dx-\dfrac{1}{3}=e-\left(e^x\right)\Big|_0^1-\dfrac{1}{3}=1-\dfrac{1}{3}=\dfrac{2}{3}$.

c) Au loc egalitățile: $\int_e^{e^2}\dfrac{f(\ln x)}{x}dx=\int_e^{e^2}(\ln x)'\cdot f(\ln x)dx\underset{u'(x)=\frac{1}{x}}{\overset{u(x)=\ln x}{=}}\int_e^{e^2}f\left(u(x)\right)\cdot u'(x)dx=$

$\underset{a\ fct.\ f}{\overset{F\ prim.}{=}}F\left(u(x)\right)\Big|_e^{e^2}=F\left(\ln e^2\right)-F(\ln e)=F(2)-F(1)$.