Soluție

1.a $f'(x)=\dfrac{1}{1+x^2}$ $\quad f''(x)=\dfrac{-2x}{(1+x^2)^2}\le 0, \forall x\in[0;\infty)$

b **L=** $\lim\limits_{x\to\infty} x^2(f(x+1)-f(x))=1$ utilizand l'Hopital sau aplicand teorema lui Lagrange

c $Fie\ g:\mathbb{R}\to\mathbb{R}, g(x)=f(x)-x+\dfrac{x^3}{3}, g'(x)=\dfrac{x^4}{1+x^2}\ge 0$

$g(0)=0, g\ \ crescatoare$ de unde $x\in(-\infty;0)$

2.a $\int_0^1 x(1+x^2)f(x)dx=\dfrac{1}{2}\ln(1+x^2)\Big|_0^1=\dfrac{1}{2}\ln 2$

b $F'(x)=x^4 f(x)$ **,** $F'(x)\ge 0$,deci F este strict crescatoare

c $A=\int_0^a f(x)dx, a<1\Rightarrow A<0<\dfrac{1}{4}$

$a\ge 1, A\le\int_1^a \dfrac{x}{(1+x^2)^2}dx=\dfrac{1}{4}-\dfrac{1}{2(1+a^2)}$

$A<\dfrac{1}{4}$