Soluție

1.a $f_a^{'}(x)=\ln\left(1+\frac{1}{x}\right)-\frac{x+a}{x(x+1)}$

b $f_a^{''}(x)=\frac{x(2a-1)+a}{x^2\left(x+1\right)^2}$

f convexa pe $(0;\infty)$daca si numai daca $f^{''}(x)\geq 0,\forall x>0 \Leftrightarrow$

$2a-1\geq 0$ si $\frac{a}{2a-1}>0$,de unde $a\in\left[\frac{1}{2};\infty\right]$

c $\lim\limits_{x\to\infty} f(x)=\infty\cdot 0$

$L=\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\frac{\ln\left(x+1\right)-\ln x}{\frac{1}{x+a}}=\frac{\infty}{\infty}$

$L=1$ folosind regula lui l'Hopital

2.a $I_2=\int_0^{\frac{\pi}{2}}\cos^2 x dx=\int_0^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}dx$

$\frac{I_n}{I_{n+1}}<\frac{n+1}{n}$ $I_2=\frac{\pi}{4}$

b $I_n=\int_0^{\frac{\pi}{2}}\cos^{n-1}x\left(\sin x\right)'dx=\left(n-1\right)\int_0^{\frac{\pi}{2}}\cos^{n-2}x\sin^2 xdx$

$=\left(n-1\right)I_{n-2}-\left(n-1\right)I_n$,de unde

$nI_n=\left(n-1\right)I_{n-2}$

c $\cos x\in\left[0;1\right]$,de unde $I_{n+1}<I_n$;in plus $I_n\geq 0$ $I_n\geq 0$,deci sirul este descrescator si marginit inferior, adica e convergent.