Soluție

a) $8x \le (x+\frac{2}{3})(3x+2)$, pentru orice $x \in [0,+\infty)$; $(3x-2)^2 \ge 0$, pentru orice $x \in [0,+\infty)$.

b) $\int f(x)dx = \int (\frac{3}{x}+\frac{1}{2\sqrt{x}}-x+\frac{1}{2})dx = 3\ln x+\sqrt{x}-\frac{x^2}{2}+\frac{x}{2}+C, C \in \mathbb{R}$.

c) f continuă pe $[-1,2]$; $vol(C_f) = \pi \int_{-1}^{2} (1-|x|)^2 dx = \pi \int_{-1}^{2} (1-2|x|+x^2)dx = \pi$.

d) $(x^3-x)\Big|_a^{a+1} = 6$; $a \in \{-2,1\}$.

e) $f,g:[1,2] \to \mathbb{R}, f(x) = \frac{x}{x+\frac{2}{3}}, g(x) = \frac{3x+2}{8}$; continue pe $[1,2]$; $f \le g$; $\int_1^2 \frac{x}{x+\frac{2}{3}}dx \le \frac{1}{8}\int_1^2 (3x+2)dx = \frac{13}{16}$.

f) f continuă, $f \ge 0$ $pe\,[1,e]$; $aria(\Gamma_f) = \int_1^e (x-1)\ln x dx = \frac{e^2-3}{4}$.