

**Soluție**

**1.a)**  $f'(x) = \frac{-4}{(x-3)^2}.$

**b)**  $\lim_{x \rightarrow 4} \frac{f(x) - f(4)}{x - 4} = f'(4) = -4.$

**c)**  $\lim_{x \rightarrow \infty} f(x) = 1 \Rightarrow y = 1$  asimptotă orizontală.

**2.a)**  $\int_0^1 f(x) dx = \ln(x+1) \Big|_0^1 = \ln 2.$

**b)**  $V = \pi \int_0^2 \frac{1}{(x+1)^2} dx = \pi \cdot \frac{-1}{x+1} \Big|_0^2 = \pi \left( \frac{-1}{3} + 1 \right) = \frac{2\pi}{3}.$

**c)**  $a \leq x \leq a+1 \Leftrightarrow a+1 \leq x+1 \leq a+2 \Leftrightarrow \frac{1}{a+2} \leq \frac{1}{x+1} \leq \frac{1}{a+1}$

$$\Rightarrow \int_a^{a+1} \frac{1}{a+2} dx \leq \int_a^{a+1} f(x) dx \leq \int_a^{a+1} \frac{1}{a+1} dx$$

$$\Leftrightarrow \frac{1}{a+2} \cdot x \Big|_a^{a+1} \leq \int_a^{a+1} f(x) dx \leq \frac{1}{a+1} \cdot x \Big|_a^{a+1} \Rightarrow \text{c.c.t.d.}$$