

Ministerul Educației, Cercetării și Tineretului
Centrul Național pentru Curriculum și Evaluare în Învățământul Preuniversitar

Rezolvare.

$$\text{a)} \quad xB + yI_3 = \begin{pmatrix} -2x & x & x \\ x & -2x & x \\ x & x & -2x \end{pmatrix} + \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix} = \begin{pmatrix} -2x+y & x & x \\ x & -2x+y & x \\ x & x & -2x+y \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} -2x+y & x & x \\ x & -2x+y & x \\ x & x & -2x+y \end{pmatrix} \Rightarrow \begin{cases} -2x+y=2 \\ x=2 \end{cases} \Rightarrow \begin{cases} x=2 \\ y=6 \end{cases}.$$

$$\text{b)} \quad A - 3I_3 = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \Rightarrow \det(A - 3I_3) = \begin{vmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{vmatrix} = 27.$$

$$\text{c)} \quad BA = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_3,$$

$$AB = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_3 \Rightarrow BA = AB \Rightarrow B \in M.$$

$$\text{d)} \quad aA \cdot A = aA^2, \quad A \cdot aA = aA^2 \Rightarrow aA \cdot A = A \cdot aA \Rightarrow aA \in M.$$

$$\text{e)} \quad B + A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} + \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{pmatrix} \Rightarrow (B + A) \cdot \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} = \begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{pmatrix} \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 3y & 3z \\ 3x & 0 & 3z \\ 3x & 3y & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \Rightarrow \begin{cases} 3y=1 \\ 3z=1 \\ 3x=1 \end{cases} \Rightarrow x = \frac{1}{3}, \quad y = \frac{1}{3}, \quad z = \frac{1}{3}.$$

$$\text{f)} \quad (X + Y)A = X \cdot A + Y \cdot A, \quad A(X + Y) = A \cdot X + A \cdot Y, \quad \text{dar } X, Y \in M \Rightarrow X \cdot A = A \cdot X, \quad Y \cdot A = A \cdot Y \Rightarrow (X + Y)A = A(X + Y) \Rightarrow X + Y \in M.$$