Soluție

1. a) $\overrightarrow{AO}=\overrightarrow{AB}+\overrightarrow{BO}$, $\overrightarrow{BO}=\overrightarrow{OD} \Rightarrow \overrightarrow{AO}=\overrightarrow{AB}+\overrightarrow{OD}$

1. b) $\overrightarrow{AO}=\overrightarrow{AB}+\overrightarrow{OD}$, $\overrightarrow{AO}=\overrightarrow{AD}+\overrightarrow{DO} \Rightarrow 2\overrightarrow{AO}=\overrightarrow{AB}+\overrightarrow{AD} \Rightarrow \overrightarrow{AO}=\dfrac{\overrightarrow{AB}+\overrightarrow{AD}}{2}$

2. a) În ΔADC: $m(\sphericalangle ADC)=90^\circ$, $m(\sphericalangle ACD)=45^\circ \Rightarrow m(\sphericalangle DAC)=45^\circ \Rightarrow AD=DC \Rightarrow AC=AD\sqrt{2}$.
$\Rightarrow AD=3\sqrt{3}$

2. b) În ΔADB: $m(\sphericalangle ADB)=90^\circ$, $m(\sphericalangle ABD)=60^\circ$, $AD=3\sqrt{3} \Rightarrow BD=\operatorname{tg}60^\circ\cdot AD=\dfrac{\sqrt{3}}{3}\cdot 3\sqrt{3}=3$.

Din punctul a) $CD=AD=3\sqrt{3} \Rightarrow BC=3\left(1+\sqrt{3}\right)$ ($D\in[BC]$, deoarece ΔABC este ascuțitunghic).

Din teorema sinusurilor avem: $\dfrac{BC}{\sin(\sphericalangle BAC)}=\dfrac{AC}{\sin(\sphericalangle ABC)} \Rightarrow \sin(\sphericalangle BAC)=\dfrac{1+\sqrt{3}}{2\sqrt{2}}=\dfrac{\sqrt{2}+\sqrt{6}}{4}$.

3. a) $y_A=y_B \Rightarrow AB \parallel Ox \Rightarrow d(C,AB)=|y_C-y_A|=3$ și $AB=|x_A-x_B|=7 \Rightarrow \mathcal{A}_{ABC}=\dfrac{3\cdot 7}{2}=\dfrac{21}{2}$

3. b) $ABCD$ paralelogram $\Rightarrow \overrightarrow{AB}=\overrightarrow{DC} \Rightarrow (x_B-x_A, y_B-y_A)=(x_C-x_D, y_C-y_D) \Rightarrow$
$(7,0)=(1-x_D, 2-y_D) \Rightarrow D(-6,2)$