

Soluție

1. a) $\frac{MC}{AM} = \frac{1}{5} \Rightarrow 5\overrightarrow{MC} = \overrightarrow{AM} \Rightarrow \overrightarrow{BM} = \frac{5\overrightarrow{BC} + \overrightarrow{BA}}{6} \Rightarrow \overrightarrow{MB} = \frac{5\overrightarrow{CB} + \overrightarrow{AB}}{6} \Rightarrow 6\overrightarrow{MB} = 5\overrightarrow{CB} + \overrightarrow{AB}.$

1. b) $\frac{NC}{DN} = \frac{1}{4} \Rightarrow \overrightarrow{DN} = 4\overrightarrow{NC} \Rightarrow \overrightarrow{BN} = \frac{\overrightarrow{BD} + 4\overrightarrow{BC}}{5}.$

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{BC} \Rightarrow \overrightarrow{BN} = \frac{\overrightarrow{BA} + \overrightarrow{BC} + 4\overrightarrow{BC}}{5} = \frac{\overrightarrow{BA} + 5\overrightarrow{BC}}{5} = -\frac{5}{6}\overrightarrow{MB} \Rightarrow B, M \text{ și } N \text{ sunt coliniare.}$$

2. a) $m(\sphericalangle KAD) = m(\sphericalangle BAD) - m(\sphericalangle KAB) \Rightarrow m(\sphericalangle KAD) = 90^\circ - 60^\circ = 30^\circ.$

2. b) În triunghiul KAD avem $m(\sphericalangle KAD) = 30^\circ$ și $KA = AD = 2$

Din teorema cosinusului avem $KD^2 = AD^2 + KA^2 - 2 \cdot KA \cdot AD \cdot \cos(\sphericalangle KAD)$

$$\Rightarrow KD^2 = 4 + 4 - 2 \cdot 2 \cdot 2 \cdot \frac{\sqrt{3}}{2} = 8 - 4\sqrt{3} \Rightarrow KD = \sqrt{6} - \sqrt{2}.$$

3. a) $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \Rightarrow AB = \sqrt{3^2 + 1^2} = \sqrt{10}.$

3. b) $AB: \frac{y - y_A}{x - x_A} = \frac{y_B - y_A}{x_B - x_A} \Rightarrow AB: \frac{y - 2}{x + 1} = \frac{1}{3} \Rightarrow AB: x - 3y + 7 = 0.$

$$d(O, AB) = \frac{|x_O - 3y_O + 7|}{\sqrt{1^2 + 3^2}} = \frac{7\sqrt{10}}{10}.$$