

Soluție

1.

a) $p^2 + p + 1 = q^2 + q - 1 \Rightarrow (p - q)(p + q + 1) = -2 \Rightarrow \begin{cases} p - q = -1 \\ p + q + 1 = 2 \end{cases} \Rightarrow p = 0 \text{ și } q = 1 \Rightarrow M = \{1\}$

b) $\frac{\lg a + \lg b}{2} = \lg \frac{a+b}{2} \Leftrightarrow (\sqrt{a} \cdot \sqrt{b}) = \frac{a+b}{2} \Leftrightarrow (\sqrt{a} - \sqrt{b})^2 = 0 \Leftrightarrow a = b$

2. $r = 3 \Rightarrow x = 1 + (n-1) \cdot 3 \Rightarrow \left[2 + (n-1) \cdot 3 \right] \frac{n}{2} = 117 \Rightarrow 3n^2 - n - 234 = 0 \Rightarrow n = 9 \Rightarrow x = 25$

3. $5C_n^3 > C_{n+1}^4, n \in \mathbb{N}, n \geq 3 \Leftrightarrow 5 \frac{n(n-1)(n-2)}{6} > \frac{(n+1)n(n-1)(n-2)}{24} \Leftrightarrow n+1 < 20$
 $\Rightarrow n \in \{3, 4, 5, \dots, 19\}$

4.

a) $x = y = 0 \Rightarrow f^3(0) = f(0) \Rightarrow f(0) \in \{0, 1\}$

b) $f(0) = 1 \Rightarrow f(x) = 1$ pentru orice $x \in \mathbb{R} \Rightarrow \text{Im } f = \{1\}$