Soluție

1. a) $\overrightarrow{PQ}=\overrightarrow{PN}+\overrightarrow{NQ}$

$\overrightarrow{PQ}=\overrightarrow{PA}+\overrightarrow{AB}+\overrightarrow{BQ}=\overrightarrow{NP}+\overrightarrow{AB}+\overrightarrow{QN} \Rightarrow 2\overrightarrow{PQ}=\overrightarrow{PN}+\overrightarrow{NQ}+\overrightarrow{NP}+\overrightarrow{AB}+\overrightarrow{QN}=\overrightarrow{AB} \Rightarrow \overrightarrow{PQ}=\frac{1}{2}\overrightarrow{AB}$

1. b) $\overrightarrow{NM}=\overrightarrow{NC}+\overrightarrow{CM}$ și $\overrightarrow{NM}=\overrightarrow{NA}+\overrightarrow{AB}+\overrightarrow{BM}=\overrightarrow{CN}+\overrightarrow{AB}+\overrightarrow{MC} \Rightarrow$

$2\overrightarrow{NM}=\overrightarrow{NC}+\overrightarrow{CM}+\overrightarrow{CN}+\overrightarrow{AB}+\overrightarrow{MC}=\overrightarrow{AB} \Rightarrow \overrightarrow{NM}=\frac{1}{2}\overrightarrow{AB} \Rightarrow \overrightarrow{PQ}=\overrightarrow{NM}$

2. a) $m(\sphericalangle ACB)=180^\circ-75^\circ-60^\circ=45^\circ$.

Din teorema sinusurilor în triunghiul ABC avem: $\frac{AC}{\sin B}=\frac{AB}{\sin C} \Rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}}=\frac{AB}{\frac{\sqrt{2}}{2}} \Rightarrow AB=2\sqrt{2}$

2. b) Din teorema cosinusului în triunghiul ABC avem $AC^2=AB^2+BC^2-2AB\cdot BC\cos B \Rightarrow$
$BC^2-2\sqrt{2}\cdot BC-4=0 \Rightarrow BC=\sqrt{2}\pm\sqrt{6}$, $BC>0 \Rightarrow BC=\sqrt{2}+\sqrt{6}$.

3. a) $d: y=-\frac{3}{4}x+3 \Rightarrow m_d=-\frac{3}{4}$

3. b) $d(O,d)=\frac{|3x_O+4y_O-12|}{\sqrt{3^2+4^2}} \Rightarrow d(O,d)=\frac{|-12|}{\sqrt{25}}=\frac{12}{5}$