Solutie

1. a) $f(x)=\dfrac{e^x-1}{x}$ *daca* $x\neq 0$ *si* $f(0)=a$; $f(1)=e-1$, $f^{'}(1)=1$ deci

$y=x+e-2$ este ecuatia tangentei

b) $\lim\limits_{x\to 0} f(x)=1$, deci f este continua in $x=0$ daca si numai daca $f(0)=a=1$

c) Evident f este derivabila pe $\mathbb{R}\setminus\{0\}$ si

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{e^x-x-1}{x^2}=\lim_{x\to 0}\frac{e^x-1}{2x}=\frac{1}{2}=f^{'}(0)$$

2.a $cu\ \mathrm{tg}\dfrac{\mathrm{x}}{2}=t$ ramane de calculat $\displaystyle\int\frac{1}{\mathrm{t}^2+2}dt$

si se obtine $\dfrac{1}{\sqrt{2}}arctg\dfrac{t}{\sqrt{2}}$

$$F(x)=\frac{1}{\sqrt{2}}arctg\frac{tg\frac{x}{2}}{\sqrt{2}}+c$$

b $F'(x)=\dfrac{1}{3+\cos x}$

$\cos x\in[-1;1]\Rightarrow F'(x)>0$

F strict crescatoare

c $\dfrac{1}{4}\leq f(x)\leq\dfrac{1}{2}\ \ \forall \mathrm{x}\in[0;\pi)$

$\dfrac{x}{4}<\displaystyle\int_0^x f(t)dt\leq\frac{x}{2}$

$\dfrac{1}{4x}<\dfrac{1}{x^2}\displaystyle\int_0^x f(t)dt\leq\frac{1}{2x}$, x>0 $\Rightarrow$ limita este egala cu 0