

Soluție

$$\text{a)} \left. \begin{array}{l} \overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} \\ \overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC} \end{array} \right\} \Rightarrow \overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB}$$

$$\text{1.b)} \left. \begin{array}{l} \overrightarrow{AB} + 2\overrightarrow{CD} + \overrightarrow{AD} = -\vec{i} + 4\vec{j} - 4\vec{i} - 16\vec{j} - 6\vec{i} - 6\vec{j} = -11\vec{i} - 18\vec{j} \\ \overrightarrow{AB} + 2\overrightarrow{CD} + \overrightarrow{AD} \text{ sunt } (-11; -18) \end{array} \right\} \Rightarrow \text{coordonatele vectorului}$$

$$\text{Din teorema sinusurilor rezultă: } \frac{BC}{\sin 60^\circ} = \frac{AC}{\sin B} \Leftrightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{\sin B} \Rightarrow \sin B = 1$$

$$\text{2.a)} \text{ Deci } m(\sphericalangle B) = 90^\circ. \text{ Atunci } \sin C = \cos A = \frac{1}{2} = \frac{AB}{AC} \Rightarrow AB = \frac{AC}{2} = 1 \Rightarrow$$

$$\text{Perimetrul } \triangle ABC \text{ este } AB + BC + AC = 3 + \sqrt{3}$$

$$\text{2.b)} AD = \frac{AB \cdot AC}{BC}, \text{ unde } AD \perp BC, D \in [BC] \Rightarrow AD = \frac{AB \cdot AC}{\sqrt{AB^2 + AC^2}} = \frac{3\sqrt{3} \cdot 3}{\sqrt{27 + 9}} = \frac{9\sqrt{3}}{\sqrt{36}} = \frac{9\sqrt{3}}{6} = \frac{3\sqrt{3}}{2}$$

$$\text{3.a)} AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$$

$$\text{3.b)} \text{ Fie } M \text{ mijlocul segmentului } [AB], M(-1, 2).$$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{2}{4} = \frac{1}{2}. \text{ Atunci ecuația mediatoarei } AM \text{ este } \frac{y - 2}{x + 1} = -2 \Rightarrow$$

$$AM : y + 2x = 0$$