

Ministerul Educației, Cercetării și Tineretului
Centrul Național pentru Curriculum și Evaluare în Învățământul Preuniversitar

Rezolvare.

$$\text{a)} \quad X(a) = \begin{pmatrix} 9 & 0 & 9 \\ 0 & 9 & 0 \\ 9 & 0 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 4 \end{pmatrix} \Rightarrow X(a) = \begin{pmatrix} 5 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 5 \end{pmatrix} \Rightarrow a = 5.$$

$$\text{b)} \quad X(-a) = \begin{pmatrix} -a & 0 & -a \\ 0 & -a & 0 \\ -a & 0 & -a \end{pmatrix} = -\begin{pmatrix} a & 0 & a \\ 0 & a & 0 \\ a & 0 & a \end{pmatrix} = -X(a).$$

$$\text{c)} \quad X(-2) + X(-1) + X(0) + X(1) + X(2) + X(3) = X(-2) + X(2) + X(-1) + X(1) + X(0) + X(3). \text{ Din b)} \\ \Rightarrow X(-2) + X(2) = X(-1) + X(1) = O_2 \Rightarrow$$

$$X(-2) + X(-1) + X(0) + X(1) + X(2) + X(3) = X(0) + X(3) = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix}.$$

$$\text{d)} \quad X(1) \cdot X(10) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 10 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 20 \end{pmatrix},$$

$$X(2) \cdot X(5) = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 20 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 20 \end{pmatrix} \Rightarrow X(1) \cdot X(10) = X(2) \cdot X(5).$$

$$\text{e)} \quad \det(X(a) + I_3) = \begin{vmatrix} a+1 & 0 & a \\ 0 & a+1 & 0 \\ a & 0 & a+1 \end{vmatrix} = (a+1)^3 - a^2(a+1) = 2a^2 + 3a + 1. \text{ Matricea } X(a) + I_3$$

$$\text{inversabilă} \Rightarrow 2a^2 + 3a + 1 \neq 0 \Rightarrow a \notin \left\{-1, -\frac{1}{2}\right\} \Rightarrow a \in \mathbb{R} - \left\{-1, -\frac{1}{2}\right\}.$$

$$\text{f)} \quad Y = \begin{pmatrix} x & y & z \\ p & q & r \\ d & e & f \end{pmatrix}, x, y, z, p, q, r, d, e, f \in \mathbb{R},$$

$$Y \cdot X(a) = \begin{pmatrix} x & y & z \\ p & q & r \\ d & e & f \end{pmatrix} \begin{pmatrix} a & 0 & a \\ 0 & a & 0 \\ a & 0 & a \end{pmatrix} = \begin{pmatrix} a(x+z) & ay & a(x+z) \\ a(p+r) & aq & a(p+r) \\ a(d+f) & ae & a(d+f) \end{pmatrix},$$

$$X(a) \cdot Y = \begin{pmatrix} a & 0 & a \\ 0 & a & 0 \\ a & 0 & a \end{pmatrix} \begin{pmatrix} x & y & z \\ p & q & r \\ d & e & f \end{pmatrix} = \begin{pmatrix} a(x+d) & a(y+e) & a(x+z) \\ ap & aq & ar \\ a(x+d) & a(y+e) & a(z+f) \end{pmatrix} \Rightarrow z = d, e = 0, r = 0, p = 0, x = f,$$

$$y = 0, y = d \Rightarrow Y = \begin{pmatrix} x & 0 & z \\ 0 & q & 0 \\ z & 0 & x \end{pmatrix}, x, d, q \in \mathbb{R}.$$