

Soluție:

1. $f\left(\frac{\alpha + \beta}{2}\right) = a \cdot \frac{\alpha + \beta}{2} + b = \frac{a(\alpha + \beta) + 2b}{2} = \frac{a\alpha + b + a\beta + b}{2} = \frac{f(\alpha) + f(\beta)}{2}.$

2. a) $\Delta \leq 0 \Leftrightarrow m^2 - 4m + 3 \leq 0 \Leftrightarrow m \in [1, 3]$

b) $(x, y) \in \{(-3, 1), (1, -3)\}.$

3. $V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \Rightarrow V\left(-\frac{1}{2}, -\frac{9}{4}\right)$ și $-\frac{1}{2} \in [-2, 1].$

$$f(-2) = f(1) = 0 \Rightarrow f([-2, 1]) = [f_{\min}, f(1)] = \left[-\frac{9}{4}, 0\right].$$

4. a) $\sqrt[3]{x} + \sqrt[3]{1-x} = 1 \Leftrightarrow x + 1 - x + 3\sqrt[3]{x-x^2} \underbrace{\left(\sqrt[3]{x} + \sqrt[3]{1-x}\right)}_1 = 1 \Leftrightarrow 3\sqrt[3]{x-x^2} = 0 \Rightarrow x \in \{0, 1\}.$

b) $2^x + 2^{x+1} + 2^{x+3} = 88 \Leftrightarrow 2^x + 2 \cdot 2^x + 8 \cdot 2^x = 88 \Leftrightarrow 11 \cdot 2^x = 88 \Leftrightarrow 2^x = 8 \Leftrightarrow x = 3.$