

**Rezolvare**

$$1.a. \text{ Din } A^2 = \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{3} & \hat{0} \\ \hat{0} & \hat{0} & \hat{5} \end{pmatrix} \cdot \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{3} & \hat{0} \\ \hat{0} & \hat{0} & \hat{5} \end{pmatrix} = \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{1} & \hat{0} \\ \hat{0} & \hat{0} & \hat{1} \end{pmatrix} = I_3.$$

$$b. A \cdot X = I_3 \Leftrightarrow \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{3} & \hat{0} \\ \hat{0} & \hat{0} & \hat{5} \end{pmatrix} \cdot \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & l \end{pmatrix} = \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{1} & \hat{0} \\ \hat{0} & \hat{0} & \hat{1} \end{pmatrix} \Leftrightarrow \begin{pmatrix} a & b & c \\ 3d & 3e & 3f \\ 5g & 5h & 5l \end{pmatrix} = \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{1} & \hat{0} \\ \hat{0} & \hat{0} & \hat{1} \end{pmatrix} \Leftrightarrow X = \begin{pmatrix} \hat{1} & \hat{0} & \hat{0} \\ \hat{0} & \hat{3} & \hat{0} \\ \hat{0} & \hat{0} & \hat{5} \end{pmatrix}.$$

$$c. \text{ Avem } (B-A)^2 = \begin{pmatrix} \hat{0} & \hat{0} & \hat{0} \\ \hat{2} & \hat{0} & \hat{0} \\ \hat{3} & \hat{7} & \hat{0} \end{pmatrix} \cdot \begin{pmatrix} \hat{0} & \hat{0} & \hat{0} \\ \hat{2} & \hat{0} & \hat{0} \\ \hat{3} & \hat{7} & \hat{0} \end{pmatrix} = \begin{pmatrix} \hat{0} & \hat{0} & \hat{0} \\ \hat{0} & \hat{0} & \hat{0} \\ \hat{6} & \hat{0} & \hat{0} \end{pmatrix}.$$

$$2.a. x * e = e * x = x \Leftrightarrow 3xe + 7x + 7e + 14 = x \Leftrightarrow e(3x + 7) = -6x - 14$$

$$\Leftrightarrow e = \frac{-6x - 14}{3x + 7} \Leftrightarrow e = \frac{-2(3x + 7)}{3x + 7} \Leftrightarrow e = -2 \text{ -element neutru.}$$

$$b. x * x \leq -\frac{7}{3} \Leftrightarrow 3x^2 + 14x + 14 \leq -\frac{7}{3} \Leftrightarrow 9x^2 + 42x + 49 \leq 0 \Leftrightarrow x = -\frac{7}{3}.$$

$$c. x * x' = x' * x = e \Leftrightarrow 3xx' + 7x + 7x' + 14 = -2 \Leftrightarrow x'(3x + 7) = -7x - 16 \Leftrightarrow x' = \frac{-7x - 16}{3x + 7}, x' \text{ există}$$

$$\text{dacă } 3x + 7 \neq 0 \Rightarrow x \neq -\frac{7}{3} \notin \mathbb{Z} \text{ și din } \Leftrightarrow x' = \frac{-7x - 16}{3x + 7}, x' \in \mathbb{Z} \Rightarrow x = -2 \text{ este singurul element inversabil.}$$