

Soluție

1.a) $a_1 > a_0$
Presupunem ca $a_{k+1} > a_k \Rightarrow a_{k+2} > a_{k+1}$

$(a_n)_{n \geq 1}$ crescator

b) $a_0 < 2, a_k < 2 \Rightarrow a_{k+1} < 2$

$(a_n)_{n \geq 1}$ marginit superior

$(a_n)_{n \geq 1}$ convergent

c)
$$l = \lim_{x \rightarrow 0} \frac{1 - \cos \pi x}{x^2} = \frac{\pi^2}{2}$$

2.a)
$$f\left(\frac{\pi}{4}\right) = \int_0^{\frac{\pi}{4}} (tg^2 t + tgt) dt$$

$$= \int_0^{\frac{\pi}{4}} ((tgt)' + tgt - 1) dt$$

$$= 1 - \frac{\pi}{4} + \frac{1}{2} \ln 2$$

b)
$$f'(x) = \frac{(\sin x + \cos x) \sin x}{\cos^2 x}, \quad \forall x \in \left(0; \frac{\pi}{2}\right)$$

$$f'(x) > 0 \Rightarrow f \text{ crescatoare}$$

c) $\alpha \geq 0, \text{ cazul } \frac{0}{0}$

Aplicand l'Hopital, $l = \lim_{x \searrow 0} \frac{(\sin x + \cos x) \sin x}{2x \cos^2 x}$

$$l = \frac{1}{2}$$