

Soluție

1.a) $f'(x) = a^x \ln a - ax^{a-1}$

$$f'(1) = a(\ln a - 1)$$

b) $f'(a) = a^a(\ln a - 1)$

$$f(a) = 0$$

$$y = a^a(\ln a - 1)(x - a)$$

c) $f(x) \geq 0 = f(a)$

$$\Rightarrow \text{conform Fermat (f derivabila), } f'(a) = 0 \Rightarrow a = e$$

$$g(x) = e^x - x^e \geq 0, \forall x > 0$$

2.a) $I_1 = \int_1^e \ln x dx$

$$= x \ln x \Big|_1^e - \int_1^e 1 dx$$

$$= e - e + 1 = 1$$

b) $I_n = \int_1^e x' \ln^n x dx = x \ln^n x \Big|_1^e - n \int_1^e \ln^{n-1} x$

$$= e - n I_{n-1}$$

c) $\ln x \in [0; 1] \Rightarrow I_{n+1} \leq I_n$

$$I_n > 0$$

$$(I_n)_{n \geq 1} \text{ descrescator si marginit} \Rightarrow (I_n)_{n \geq 1} \text{ convergent}$$